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CHEMICAL ARITHMETIC.

PART I.

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CHEMICAL ARITHMETIC.

PART I.

A COLLECTION OF TABLES,
MATHEMATICAL, CHEMICAL, AND PHYSICAL, FOR THE
USE OF CHEMISTS AND OTHERS.

BY

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PREFACE.

THIS book, when completed, will consist of two parts. The first is a collection of tables and formulæ intended for the use of Chemists and others; the second an exposition of the applications of arithmetic to the solution of chemical problems.

Part I., which is embodied in the present volume, may be viewed as a new edition of my "*Tables to facilitate Chemical Calculations*," but the changes and additions are so numerous that it is more correct to call it a new book.

The general idea by which I have been guided in its compilation is to try to offer to the scientific as well as to the technical Chemist, a fairly complete collection of what he needs of mathematical auxiliaries, and of chemical and physical constants, in the ordinary routine of his laboratory work, and that *without going further*. This book, accordingly, does not pretend to compete with works of the order of Clarke's "Constants of Nature" or even of Landolt and Börnstein's "Tabellen," nor is it intended to supplant books which, like Lunge and Hurter's "Pocket-Book," address themselves to specialists in this or that branch of technical chemistry. From the majority of the now numerous works of a similar tendency it differs chiefly in this, that it is not a mere compilation but, essentially, an original production. There are, indeed, a few tables, and a few only, in it which were simply copied from other books, but amongst these transferences there is, I trust, not one which could be called a piracy.

What I have said applies to some extent to even the logarithm tables given between pages 2 and 47. The logarithms themselves were, of course, simply transcribed from standard works, but each of the tables exhibits certain original features, which, I venture to hope, will be recognized as improvements on the ordinary system.

The *Five-Place Table* differs from others in this, that it is continued up to the *numerus* 14 999, which does away with the troublesome interpolations in connection with *numeri* between 10 000 and 15 000; and the interpolations generally are greatly facilitated by a novel arrangement of the marginal tables of proportional parts ("PP's") which consists in this, that instead of the increment in the number, $\Delta \text{num.}$, that of the logarithm, $\Delta \text{log.}$, is taken as the independent variable, and that every integer value of $\Delta \text{log.}$ presents itself in the respective column. In seeking for the logarithm of a number not given directly by the table, find the nearest tabular Δn , and adopt the value $\Delta \text{log.}$ standing opposite to it. If the number for a given logarithm has to be found, any value $\Delta \text{log.}$ that can possibly present itself is found in the second PP-column, and opposite to it stands the corresponding Δn calculated to one decimal-place. Thus all interpolations can be done at sight, and this with the highest attainable precision. The table, indeed, may be said to combine the advantages of a logarithm and an "anti-logarithm" table with this important difference in its favour, that the precision of the number found is the same throughout the whole of the cycle. (In ordinary five-place anti-logarithm tables the *numeri* are given to five places only, which obviously does not do justice to any mantissa lying between .000 00 and about .6). The graphic representation of the PP's for the logarithmic intervals 44 to 29 (on page 19) is an experiment, the practical issue of which remains to be seen. All I can say, meanwhile, is that the diagrams have been tested by a series of critical readings and been found to be sufficiently correct. Quick arithmeticians may perhaps rather use page 18 with these graphic PP's than the part of the table which goes from 10 000 to 15 000. The use of the diagrams, of course, is optional, and, at the worst, they stand in nobody's way. In the arrangement of the table I have followed Bremiker in so far that, like him, I let every page begin with an integer multiple of 50. Another improvement of Bremiker's, which is that for every logarithm the *four* last figures are given in its *tabular locus*, I could not have adopted without enlarging my pages, which I did not feel inclined to do, the less so as the advantage of Bremiker's arrangement, which is to reduce the number of changes in the separated figures, is not so great in a table going from 2 000 to 15 000 as it is in one which stops at 10 000. I preferred to adopt the usual system, which is to separate only the first two figures, *i.e.*, print each set of these only once where it appears for the first time in the "o"-column. Of the many devices which have been invented for showing where the change comes in, that of Sang is undoubtedly the best. It consists in the substitution of a "notka" ♦ for any o appearing as a third figure in the

respective line-section. I accordingly have adopted it, and, as in a five-place table the notka may appear in only one or two of the cases where it would be needed (if the third figure were a 0), I have caused the whole of the respective set of entries to be printed in "*black type*."

To pass to the *Four-Place Table*. I originally intended to reproduce that single-entry table which stands at the beginning of my "*Tables to facilitate*," and was the more inclined to do so, as this table, in the course of a number of years' constant use, had proved very convenient; but, with the less size of page adopted for the present edition, it would have taken up too many pages, and, on this account, have been less handy. After some hesitation I decided upon rejecting it and substituting an ordinary double-entry table, taking care however to continue this to 2000, and to introduce my own system of PP's, although these, for the interval from the num. 200 to the num. 500, demand a whole page for themselves.

I do not approve of that now popular kind of four-place table in which the PP's for $\Delta \text{ num.} = 1, 2, 3, 4, \dots 9$, are given in a line with respective line of ten logarithms, and calculated, as integers, on the assumption that, even for the numerical interval 1000 to 1100, the ratio $\frac{\Delta \log.}{\Delta \text{ num.}}$ is constant. With such a table the uncertainty of a logarithm found by interpolation rises to $\pm .00025$, corresponding to an error in the number equal to about $1/16000$ th of its value, while in a properly constructed four-place table the corresponding uncertainties are $.0001$ and $1/4000$ th.

In addition to the five- and four-place, I give two *Three-Place Logarithm Tables*, distinguished as "No. 1 and No. 2." Table No. 1 is a single-entry table constructed on precisely the same principle as the now abolished four-place table of the "*Tables to facilitate*" was; in No. 2, all three-place mantissæ from .000 to .999 are enumerated in column II., and, opposite to each, stands the respective *numerus* calculated to four figures. It is, indeed, nothing more or less than an ordinary four-place anti-logarithm table with the variables reversed, and, of course, may be used in this capacity. But it is *meant* to be used as a three-place logarithm table *demanding no interpolation*. As may be seen at a glance, any single logarithm taken out of the table cannot be wrong by more than $\pm .0005$, corresponding to $1/800$ th of the value of the respective number, hence the table, in point of precision, is fully at a par with the majority of ordinary analytical determinations.

In a book which, like the present, contains an *indexed* five-place table, both the four- and the three-place table might appear to be

superfluous. A good many years ago, when I had just realised the idea of indexing the five-place table which I was then in the habit of using, I thought so myself, but I have come to see that I was mistaken. For anyone who cares it is easy to put the matter to the test. All he needs to do is to carry out (say) fifty calculations with four-place logarithms, once with our four-place table, and once with our five-place one used as four-place. In most hands the five-place will come off second best. I am not so sure if our four-place table used as a three-place would not work more expeditiously than our three-place No. 2; the latter, however, will always retain its value as an occasionally useful four-place anti-logarithm table. Our three-place table No. 1 will not perhaps appear in the next edition of this book; but it so happened that it was printed and corrected before No. 2 was thought of, and as it fills only two pages, and is useful for teaching purposes, I allowed it to stand.

The *Reciprocal Table* is simply a reprint of the one which formed part of the old edition; but, in correcting the proofs, instead of merely following that edition, I re-read all the reciprocals with Barlow's table, and, I am glad to say, found no error. A four-place table of reciprocals is a little ahead of a four-place table of logarithms in point of precision, and it is obviously the more convenient table of the two whenever the reciprocal is wanted for its own sake. Now, such cases occur nowhere more frequently than in practical chemistry. To give only one example, and there are plenty of analogous cases. Supposing we have found that 1 CC of standard solution A neutralises x CC of solution B, and we want to know the number of CC of A which corresponds to 1 CC of B. The reciprocal table gives the answer quite directly, and on this account more exactly than the logarithm table does.

The table of values of the *Probability-Integral* and my own tables for the *Conversion of Decimals into Integer-ratios* will be welcome, if not to many, at least to a minority of my *confrères*; at anyrate, neither of the two needs apologize for its appearance in the book.

The "*Formula Values*" and "*Analytical Factors*" have all been calculated expressly for this book on the basis of the table of atomic weights given on page 1. Each calculation has been done at least twice, and this by different calculators working independently of each other. The proofs were read, finally, not with the manuscript after which they had been set up, but with the records of the respective calculations. Either table might, of course, have been spun out to any length, but I thought I must, in this case more than in any other, confine myself to what is likely to be of service to the Chemist in his

every-day practice. In my opinion the tables as they now stand are probably a little too complete. The factors for gas-volumetric determinations are based on Regnault's direct determinations of the respective gas-densities and coefficients of expansion; not that I thought, thereby, to add to the real precision of the factors as such, but it so happened that at the time the respective constants had been already calculated for the section on gasometry, and I therefore had no occasion to use the customary approximate formulæ.

The section on the "*Metric and British Systems of Units*" is based on Mr. Chaney's well-known tables in card form; but the majority of precisely those factors, which I deem to be particularly useful to the Chemist, have been calculated by myself from the fundamental data supplied by the tables referred to. The factors for the conversion of Pounds Sterling into Marks or Francs, and *vice versa*, and those for the mutual conversion of the price in Shillings or Pounds of the pound Avoirdupois and the price of the Kilogramme in Marks or Francs, etc., will be welcome to all those who, like myself, are in the habit of procuring their chemicals largely from German or French houses.

In the section on the "*Relation in certain Solutions between Percentage of Dissolved Matter and Specific Gravity*" I confined myself to the few cases which are of importance to the general Chemist. A complete collection of the respective tables would, by themselves, have filled a larger volume than the present. Two of the tables on sulphuric acid are calculated by myself after Bineau's direct determinations, and a similar remark applies to the tables on aqueous ethyl-alcohols, which latter are based on Mendelejeff's determinations. The two tables for the conversion of percentages of alcohol by volume into percentages by weight, and *vice versa*, meet a want felt, I am sure, by many Chemists besides myself, and will, I hope, be found useful.

Of the several sections of this book those on "*Calibration*" and "*Gasometry*" are, I believe, most entitled to the claim of originality. The tables giving the vapour-tensions of water and alcohol, and those embodying Bunsen's coefficients of absorption of gases are, as far as I can remember just now, while writing this preface, the only portions of these sections which have been copied from other books. All the rest is original, so far at least as arrangement is concerned. In the table giving the "*Physical Constants of some Gases according to Regnault*," the fundamental values for the gas-densities (*i.e.* the weight of a litre of air and the specific gravities on the air-scale of the other gases included) were taken directly from Lasch's memoir on his re-calculation of Regnault's determinations; the coefficients of expansion were copied

from Landolt and Börnstein's "Tabellen," and Baron Wrede's specific gravity of carbonic oxide from Berzelius' "Jahresbericht." For all the rest of the entries I am myself responsible.

In the section of "*Gasometry*" I reproduce my own methods for formulating the relations in perfect gases between weight and volume, and for the calculation and interpretation of gas-analyses, which I expounded many years ago in my article "*Analyse, volumetrische, für Gase*" in Fehling's "*Handwörterbuch*." Neither of the two has, as far as I can see, been much heeded by Chemists outside of my own Laboratory. But, as it so happens that two chemical philosophers of world-wide fame have, however unconsciously, reproduced one of them the one, and the other the other of my notions, and have found it worth while to publish them in scientific journals, I venture to hope that my methods will now become more popular. From the fact that I have left my ideas shut up in "*Handwörterbuch*" for so many years, it is clear that I am far from attaching to them more than a certain amount of didactic value.

I must not conclude without tendering my cordial thanks to Messrs. James Robson, William Cullen, J. B. Henderson, and J. F. Ness (all Assistants or Senior Students in my Laboratory), for the unselfish and zealous way in which they have assisted me in the calculation of the formula-values and analytical factors, and in the compilation of the logarithm tables. In order, however, to do full justice to Mr. Robson, I must acknowledge, specially, that he cheerfully assisted me in the reading of all the proofs of the logarithm tables—a kind of work which, it is true, is not difficult, but tiresome to a degree of which only he who has tried it can form an adequate conception. Having thus come back to the subject of the logarithm tables, I may state that we always used Sang's excellent seven-place table as our final standard, and, of course, did not rest before we were absolutely convinced that every one of our own entries agreed with the corresponding one in that classical work. In all those cases, however, where the seven-place logarithm ends in 50, we used Schrön's table for seeing whether the 50 is a genuine 50 or stands as an approximation for 49 plus more than half-a-unit of the seventh place.

The diagrams on pages 19 and 164 are the work of Mr. Ness, who constructed them most carefully by means of a little dividing-engine of my own contriving, which we are in the habit of using for the graduation of burettes, etc. Mr. Ness's drawings were photo-zincographed by Messrs. Ford & Wall of London, and thus made available for the printing press.

I am fully convinced that my tables of logarithms, and also my table of reciprocals, are free of errors; but I am not sufficiently bold to venture upon a similar statement in regard to the tables generally, although I have done my best towards making them all correct. Whoever may detect errors will earn my gratitude by communicating them to me.

W. DITTMAR.

ANDERSON'S COLLEGE, GLASGOW,
July, 1890.

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ATOMIC WEIGHTS.

$$O = 16$$

After Lothar Meyer and Seubert's calculations. Exceptions: according to authorities quoted in the notes.

Name of Element.	Atomic Weight.		Name of Element.	Atomic Weight.	
	Symbol.	Value.		Symbol.	Value.
Aluminium,	Al	27.10	Molybdenum	Mo	96.2
Antimony, -	Sb	119.9	Nickel,	Ni	58.7
Arsenic, -	As	75.09	Niobium,	Nb	93.9
Barium, -	Ba	137.20	Nitrogen, -	N	14.046
Beryllium, -	Be	9.1	Osmium, ⁷	Os	191.5
Bismuth, ¹ -	Bi	208.0	Oxygen, -	O	16.
Boron, -	B	10.9	Palladium, -	Pd	106.6
Bromine, -	Br	79.952	Phosphorus,	P	31.04
Cadmium, -	Cd	112.0	Platinum, ⁸ -	Pt	195.5
Cæsium, -	Cs	133.0	Potassium, -	K	39.136
Calcium, -	Ca	40.02	Rhodium, -	Rh	104.3
Carbon, -	C	12.00	Rubidium, -	Rb	85.4
Cerium, -	Ce	141.5	Ruthenium,	Ru	103.8
Chlorine, -	Cl	35.454	Scandium, -	Sc	44.1
Chromium, ³	Cr	52.13	Selenium, -	Se	79.07
Cobalt, -	Co	58.7	Silicon, ⁹ -	Si	28.40
Copper, -	Cu	63.34	Silver, -	Ag	107.93
Didymium, -	Di	145.4	Sodium, -	Na	23.053
Erbium, -	E	166.4	Strontium, -	Sr	87.52
Fluorine, -	F	19.1	Sulphur, -	S	32.06
Gallium,	Ga	70.1	Tantalum, -	Ta	182.7
Gold, ⁴ -	Au	197.22	Tellurium, -	Te	128.0
Hydrogen, -	H	1.0024	Thallium, -	Tl	204.2
Indium, -	In	113.7	Thorium, -	Th	232.5
Iridium, -	Ir	193.0	Tin, -	Sn	117.6
Iodine, -	I	126.85	Titanium, ¹⁰ -	Ti	48.08
Iron, -	Fe	56.02	Tungsten, -	W	184.0
Lanthanum,	La	138.9	Uranium, -	U	240.5
Lead, -	Pb	206.9	Vanadium,	V	51.2
Lithium, ⁵ -	Li	6.89	Yttrium, -	Y	89.8
Magnesium, ²	Mg	24.37	Ytterbium, -	Yb	173.
Manganese, ⁶	Mn	55.0	Zinc, ¹¹ -	Zn	65.37
Mercury, -	Hg	200.3	Zirconium, -	Zr	90.6

¹ and ² Marignac, 1884. ³ Siewert, as calculated by Clarke. ⁴ Mean of determinations by Thorpe and Laurie, 1887, and Krüss, 1887. ⁵ W. Dittmar, analyses of the Carbonate; R.S.E. Trans. 1889. ⁶ Dewar and Scott; Marignac. ⁷ Seubert, 1888. ⁸ Dittmar and MacArthur, 1886-7. ⁹ Thorpe and Young, 1887. ¹⁰ Thorpe, Roy. Soc. Proc., 1883. ¹¹ Mean of determinations by Marignac and by Baubigny, 1884.

N.	Log.	P.P.	N.	Log.	P.P.	N.	Log.	P.P.	N.	Log.	P.P.
100	.000		150	.176		200	.301		250	.398	
101	.004		151	.179		201	.303		251	.400	
102	.009		152	.182		202	.305		252	.401	
103	.013		153	.185		203	.307		253	.403	
104	.017		154	.188		204	.310		254	.405	
105	.021		155	.190		205	.312		255	.407	
106	.025		156	.193		206	.314		256	.408	
107	.029	Δn	157	.196		207	.316		257	.410	
108	.033	$\Delta 1$	158	.199		208	.318		258	.412	
109	.037	0.2	159	.201		209	.320		259	.413	
		0.4									
110	.041	0.6	160	.204		210	.322		260	.415	
111	.045	0.8	161	.207		211	.324		261	.417	
112	.049	1.0	162	.210		212	.326		262	.418	
113	.053		163	.212		213	.328		263	.420	
114	.057		164	.215		214	.330		264	.422	
115	.061		165	.217	Δn	215	.332	Δn	265	.423	Δn
116	.064		166	.220	$\Delta 1$	216	.334	$\Delta 1$	266	.425	$\Delta 1$
117	.068		167	.223	0.7	217	.336	0.5	267	.427	0.5
118	.072		168	.225	0.3	218	.338	1.0	268	.428	1.0
119	.076		169	.228	1.0	219	.340		269	.430	
120	.079		170	.230		220	.342		270	.431	
121	.083		171	.233		221	.344		271	.433	
122	.086		172	.236		222	.346		272	.435	
123	.090	Δn	173	.238		223	.348		273	.436	
124	.093	$\Delta 1$	174	.241		224	.350		274	.438	
		0.25									
125	.097	0.50	175	.243		225	.352		275	.439	
126	.100	0.75	176	.246		226	.354		276	.441	
127	.104	1.00	177	.248		227	.356		277	.442	
128	.107		178	.250		228	.358		278	.444	
129	.111		179	.253		229	.360		279	.446	
130	.114		180	.255	Δn	230	.362	Δn	280	.447	Δn
131	.117		181	.258	$\Delta 1$	231	.364	$\Delta 1$	281	.449	$\Delta 1$
132	.121		182	.260		232	.365		282	.450	
133	.124		183	.262	0.5	233	.367	1.0	283	.452	1.0
134	.127		184	.265	1.0	234	.369		284	.453	
135	.130		185	.267		235	.371		285	.455	
136	.134		186	.270		236	.373		286	.456	
137	.137		187	.272		237	.375		287	.458	
138	.140	Δn	188	.274		238	.377		288	.459	
139	.143	$\Delta 1$	189	.276		239	.378		289	.461	
		0.3									
140	.146	0.7	190	.279		240	.380		290	.462	
141	.149	1.0	191	.281		241	.382		291	.464	
142	.152		192	.283		242	.384		292	.465	
143	.155		193	.286		243	.386		293	.467	
144	.158		194	.288		244	.387		294	.468	
145	.161		195	.290		245	.389		295	.470	
146	.164		196	.292		246	.391		296	.471	
147	.167		197	.294		247	.393		297	.473	
148	.170		198	.297		248	.394		298	.474	
149	.173		199	.299		249	.396		299	.476	
150	.176		200	.301		250	.398		300	.477	

THREE-PLACE LOGARITHMS—TABLE NO. 1.

3

N.	Log.	P.P.	N.	Log.	P.P.	N.	Log.	P.P.	N.	Log.	P.P.
300	.477		400	.602		500	.699		750	.875	
302	.480		402	.604		505	.703		755	.878	
304	.483		404	.606		510	.708		760	.881	
306	.486		406	.609		515	.712		765	.884	
308	.489		408	.611		520	.716		770	.886	
310	.491		410	.613		525	.720		775	.889	
312	.494		412	.615		530	.724		780	.892	
314	.497		414	.617		535	.728	Δn	785	.895	
316	.500		416	.619		540	.732	Δl	790	.898	
318	.502		418	.621		545	.736	1.0	795	.900	
320	.505		420	.623		550	.740	2.0			
322	.508		422	.625		555	.744	3.0	800	.903	
324	.511		424	.627		560	.748	4.0	805	.906	
326	.513		426	.629		565	.752	5.0	810	.908	
328	.516		428	.631		570	.756		815	.911	
330	.519	Δn	430	.633	Δn	575	.760		820	.914	
332	.521	Δl	432	.635	Δl	580	.763		825	.916	Δn
334	.524	0.7	434	.637	0.7	585	.767		830	.919	Δl
336	.526	1.3	436	.639	1.3	590	.771		835	.922	1.7
338	.529	2.0	438	.641	2.0	595	.775		840	.924	3.3
340	.531		440	.643		600	.778		845	.927	5.0
342	.534		442	.645		605	.782				
344	.537		444	.647		610	.785		850	.929	
346	.539		446	.649		615	.789	Δn	855	.932	
348	.542		448	.651		620	.792	Δl	860	.934	
350	.544		450	.653		625	.796	1.3	865	.937	
352	.547		452	.655		630	.799	2.5	870	.940	
354	.549		454	.657		635	.803	3.8			
356	.551		456	.659		640	.806	5.0	875	.942	
358	.554		458	.661		645	.810		880	.944	
360	.556	Δn	460	.663	Δn	650	.813		885	.947	
362	.559	Δl	462	.665	Δl	655	.816		890	.949	
364	.561		464	.667		660	.820		895	.952	
366	.563	1.0	466	.668	1.0	665	.823		900	.954	Δn
368	.566	2.0	468	.670	2.0	670	.826		905	.957	Δl
370	.568		470	.672		675	.829		910	.959	
372	.571		472	.674		680	.833		915	.961	2.5
374	.573		474	.676		685	.836		920	.964	5.0
376	.575		476	.678		690	.839	Δn	925	.966	
378	.577		478	.679		695	.842	Δl	930	.968	
380	.580		480	.681		700	.845	1.7	935	.971	
382	.582		482	.683		705	.848	3.3	940	.973	
384	.584		484	.685		710	.851	5.0	945	.975	
386	.587		486	.687		715	.854				
388	.589		488	.688		720	.857		950	.978	
390	.591		490	.690		725	.860		955	.980	
392	.593		492	.692		730	.863		960	.982	
394	.595		494	.694		735	.866		965	.985	
396	.598		496	.695		740	.869		970	.987	
398	.600		498	.697		745	.872		975	.989	
400	.602		500	.699		750	.875		980	.991	
									985	.993	
									990	.996	
									995	.998	
									1000	.000	

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
100.0	.000	112.2	.050	125.9	.100	141.3	.150	158.5	.200
100.2	.001	112.5	.051	126.2	.101	141.6	.151	158.9	.201
100.5	.002	112.7	.052	126.5	.102	141.9	.152	159.2	.202
100.7	.003	113.0	.053	126.8	.103	142.2	.153	159.6	.203
100.9	.004	113.2	.054	127.1	.104	142.6	.154	160.0	.204
101.2	.005	113.5	.055	127.4	.105	142.9	.155	160.3	.205
101.4	.006	113.8	.056	127.6	.106	143.2	.156	160.7	.206
101.6	.007	114.0	.057	127.9	.107	143.5	.157	161.1	.207
101.9	.008	114.3	.058	128.2	.108	143.9	.158	161.4	.208
102.1	.009	114.6	.059	128.5	.109	144.2	.159	161.8	.209
102.3	.010	114.8	.060	128.8	.110	144.5	.160	162.2	.210
102.6	.011	115.1	.061	129.1	.111	144.9	.161	162.6	.211
102.8	.012	115.3	.062	129.4	.112	145.2	.162	162.9	.212
103.0	.013	115.6	.063	129.7	.113	145.5	.163	163.3	.213
103.3	.014	115.9	.064	130.0	.114	145.9	.164	163.7	.214
103.5	.015	116.1	.065	130.3	.115	146.2	.165	164.1	.215
103.8	.016	116.4	.066	130.6	.116	146.6	.166	164.4	.216
104.0	.017	116.7	.067	130.9	.117	146.9	.167	164.8	.217
104.2	.018	116.9	.068	131.2	.118	147.2	.168	165.2	.218
104.5	.019	117.2	.069	131.5	.119	147.6	.169	165.6	.219
104.7	.020	117.5	.070	131.8	.120	147.9	.170	166.0	.220
105.0	.021	117.8	.071	132.1	.121	148.3	.171	166.3	.221
105.2	.022	118.0	.072	132.4	.122	148.6	.172	166.7	.222
105.4	.023	118.3	.073	132.7	.123	148.9	.173	167.1	.223
105.7	.024	118.6	.074	133.0	.124	149.3	.174	167.5	.224
105.9	.025	118.9	.075	133.4	.125	149.6	.175	167.9	.225
106.2	.026	119.1	.076	133.7	.126	150.0	.176	168.3	.226
106.4	.027	119.4	.077	134.0	.127	150.3	.177	168.7	.227
106.7	.028	119.7	.078	134.3	.128	150.7	.178	169.0	.228
106.9	.029	119.9	.079	134.6	.129	151.0	.179	169.4	.229
107.2	.030	120.2	.080	134.9	.130	151.4	.180	169.8	.230
107.4	.031	120.5	.081	135.2	.131	151.7	.181	170.2	.231
107.6	.032	120.8	.082	135.5	.132	152.1	.182	170.6	.232
107.9	.033	121.1	.083	135.8	.133	152.4	.183	171.0	.233
108.1	.034	121.3	.084	136.1	.134	152.8	.184	171.4	.234
108.4	.035	121.6	.085	136.5	.135	153.1	.185	171.8	.235
108.6	.036	121.9	.086	136.8	.136	153.5	.186	172.2	.236
108.9	.037	122.2	.087	137.1	.137	153.8	.187	172.6	.237
109.1	.038	122.5	.088	137.4	.138	154.2	.188	173.0	.238
109.4	.039	122.7	.089	137.7	.139	154.5	.189	173.4	.239
109.6	.040	123.0	.090	138.0	.140	154.9	.190	173.8	.240
109.9	.041	123.3	.091	138.4	.141	155.2	.191	174.2	.241
110.2	.042	123.6	.092	138.7	.142	155.6	.192	174.6	.242
110.4	.043	123.9	.093	139.0	.143	156.0	.193	175.0	.243
110.7	.044	124.2	.094	139.3	.144	156.3	.194	175.4	.244
110.9	.045	124.5	.095	139.6	.145	156.7	.195	175.8	.245
111.2	.046	124.7	.096	140.0	.146	157.0	.196	176.2	.246
111.4	.047	125.0	.097	140.3	.147	157.4	.197	176.6	.247
111.7	.048	125.3	.098	140.6	.148	157.8	.198	177.0	.248
111.9	.049	125.6	.099	140.9	.149	158.1	.199	177.4	.249
112.2	.050	125.9	.100	141.3	.150	158.5	.200	177.8	.250

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
177.8	.250	199.5	.300	223.9	.350	251.2	.400	281.8	.450
178.2	.251	200.0	.301	224.4	.351	251.8	.401	282.5	.451
178.6	.252	200.4	.302	224.9	.352	252.3	.402	283.1	.452
179.1	.253	200.9	.303	225.4	.353	252.9	.403	283.8	.453
179.5	.254	201.4	.304	225.9	.354	253.5	.404	284.4	.454
179.9	.255	201.8	.305	226.5	.355	254.1	.405	285.1	.455
180.3	.256	202.3	.306	227.0	.356	254.7	.406	285.8	.456
180.7	.257	202.8	.307	227.5	.357	255.3	.407	286.4	.457
181.1	.258	203.2	.308	228.0	.358	255.9	.408	287.1	.458
181.6	.259	203.7	.309	228.6	.359	256.4	.409	287.7	.459
182.0	.260	204.2	.310	229.1	.360	257.0	.410	288.4	.460
182.4	.261	204.6	.311	229.6	.361	257.6	.411	289.1	.461
182.8	.262	205.1	.312	230.1	.362	258.2	.412	289.7	.462
183.2	.263	205.6	.313	230.7	.363	258.8	.413	290.4	.463
183.7	.264	206.1	.314	231.2	.364	259.4	.414	291.1	.464
184.1	.265	206.5	.315	231.7	.365	260.0	.415	291.7	.465
184.5	.266	207.0	.316	232.3	.366	260.6	.416	292.4	.466
184.9	.267	207.5	.317	232.8	.367	261.2	.417	293.1	.467
185.4	.268	208.0	.318	233.3	.368	261.8	.418	293.8	.468
185.8	.269	208.4	.319	233.9	.369	262.4	.419	294.4	.469
186.2	.270	208.9	.320	234.4	.370	263.0	.420	295.1	.470
186.6	.271	209.4	.321	235.0	.371	263.6	.421	295.8	.471
187.1	.272	209.9	.322	235.5	.372	264.2	.422	296.5	.472
187.5	.273	210.4	.323	236.0	.373	264.9	.423	297.2	.473
187.9	.274	210.9	.324	236.6	.374	265.5	.424	297.9	.474
188.4	.275	211.3	.325	237.1	.375	266.1	.425	298.5	.475
188.8	.276	211.8	.326	237.7	.376	266.7	.426	299.2	.476
189.2	.277	212.3	.327	238.2	.377	267.3	.427	299.9	.477
189.7	.278	212.8	.328	238.8	.378	267.9	.428	300.6	.478
190.1	.279	213.3	.329	239.3	.379	268.5	.429	301.3	.479
190.5	.280	213.8	.330	239.9	.380	269.2	.430	302.0	.480
191.0	.281	214.3	.331	240.4	.381	269.8	.431	302.7	.481
191.4	.282	214.8	.332	241.0	.382	270.4	.432	303.4	.482
191.9	.283	215.3	.333	241.5	.383	271.0	.433	304.1	.483
192.3	.284	215.8	.334	242.1	.384	271.6	.434	304.8	.484
192.8	.285	216.3	.335	242.7	.385	272.3	.435	305.5	.485
193.2	.286	216.8	.336	243.2	.386	272.9	.436	306.2	.486
193.6	.287	217.3	.337	243.8	.387	273.5	.437	306.9	.487
194.1	.288	217.8	.338	244.3	.388	274.2	.438	307.6	.488
194.5	.289	218.3	.339	244.9	.389	274.8	.439	308.3	.489
195.0	.290	218.8	.340	245.5	.390	275.4	.440	309.0	.490
195.4	.291	219.3	.341	246.0	.391	276.1	.441	309.7	.491
195.9	.292	219.8	.342	246.6	.392	276.7	.442	310.5	.492
196.3	.293	220.3	.343	247.2	.393	277.3	.443	311.2	.493
196.8	.294	220.8	.344	247.7	.394	278.0	.444	311.9	.494
197.2	.295	221.3	.345	248.3	.395	278.6	.445	312.6	.495
197.7	.296	221.8	.346	248.9	.396	279.3	.446	313.3	.496
198.2	.297	222.3	.347	249.5	.397	279.9	.447	314.1	.497
198.6	.298	222.8	.348	250.0	.398	280.5	.448	314.8	.498
199.1	.299	223.4	.349	250.6	.399	281.2	.449	315.5	.499
199.5	.300	223.9	.350	251.2	.400	281.8	.450	316.2	.500

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
316.2	.500	354.8	.550	398.1	.600	446.7	.650	501.2	.700
317.0	.501	355.6	.551	399.0	.601	447.7	.651	502.3	.701
317.7	.502	356.5	.552	399.9	.602	448.7	.652	503.5	.702
318.4	.503	357.3	.553	400.9	.603	449.8	.653	504.7	.703
319.2	.504	358.1	.554	401.8	.604	450.8	.654	505.8	.704
319.9	.505	358.9	.555	402.7	.605	451.9	.655	507.0	.705
320.6	.506	359.7	.556	403.6	.606	452.9	.656	508.2	.706
321.4	.507	360.6	.557	404.6	.607	453.9	.657	509.3	.707
322.1	.508	361.4	.558	405.5	.608	455.0	.658	510.5	.708
322.8	.509	362.2	.559	406.4	.609	456.0	.659	511.7	.709
323.6	.510	363.1	.560	407.4	.610	457.1	.660	512.9	.710
324.3	.511	363.9	.561	408.3	.611	458.1	.661	514.0	.711
325.1	.512	364.8	.562	409.3	.612	459.2	.662	515.2	.712
325.8	.513	365.6	.563	410.2	.613	460.3	.663	516.4	.713
326.6	.514	366.4	.564	411.1	.614	461.3	.664	517.6	.714
327.3	.515	367.3	.565	412.1	.615	462.4	.665	518.8	.715
328.1	.516	368.1	.566	413.0	.616	463.4	.666	520.0	.716
328.9	.517	369.0	.567	414.0	.617	464.5	.667	521.2	.717
329.6	.518	369.8	.568	415.0	.618	465.6	.668	522.4	.718
330.4	.519	370.7	.569	415.9	.619	466.7	.669	523.6	.719
331.1	.520	371.5	.570	416.9	.620	467.7	.670	524.8	.720
331.9	.521	372.4	.571	417.8	.621	468.8	.671	526.0	.721
332.7	.522	373.3	.572	418.8	.622	469.9	.672	527.2	.722
333.4	.523	374.1	.573	419.8	.623	471.0	.673	528.4	.723
334.2	.524	375.0	.574	420.7	.624	472.1	.674	529.7	.724
335.0	.525	375.8	.575	421.7	.625	473.2	.675	530.9	.725
335.7	.526	376.7	.576	422.7	.626	474.2	.676	532.1	.726
336.5	.527	377.6	.577	423.6	.627	475.3	.677	533.3	.727
337.3	.528	378.4	.578	424.6	.628	476.4	.678	534.6	.728
338.1	.529	379.3	.579	425.6	.629	477.5	.679	535.8	.729
338.8	.530	380.2	.580	426.6	.630	478.6	.680	537.0	.730
339.6	.531	381.1	.581	427.6	.631	479.7	.681	538.3	.731
340.4	.532	381.9	.582	428.5	.632	480.8	.682	539.5	.732
341.2	.533	382.8	.583	429.5	.633	481.9	.683	540.8	.733
342.0	.534	383.7	.584	430.5	.634	483.1	.684	542.0	.734
342.8	.535	384.6	.585	431.5	.635	484.2	.685	543.3	.735
343.6	.536	385.5	.586	432.5	.636	485.3	.686	544.5	.736
344.3	.537	386.4	.587	433.5	.637	486.4	.687	545.8	.737
345.1	.538	387.3	.588	434.5	.638	487.5	.688	547.0	.738
345.9	.539	388.2	.589	435.5	.639	488.7	.689	548.3	.739
346.7	.540	389.0	.590	436.5	.640	489.8	.690	549.5	.740
347.5	.541	389.9	.591	437.5	.641	490.9	.691	550.8	.741
348.3	.542	390.8	.592	438.5	.642	492.0	.692	552.1	.742
349.1	.543	391.7	.593	439.5	.643	493.2	.693	553.4	.743
349.9	.544	392.6	.594	440.6	.644	494.3	.694	554.6	.744
350.8	.545	393.6	.595	441.6	.645	495.5	.695	555.9	.745
351.6	.546	394.5	.596	442.6	.646	496.6	.696	557.2	.746
352.4	.547	395.4	.597	443.6	.647	497.7	.697	558.5	.747
353.2	.548	396.3	.598	444.6	.648	498.9	.698	559.8	.748
354.0	.549	397.2	.599	445.7	.649	500.0	.699	561.0	.749
354.8	.550	398.1	.600	446.7	.650	501.2	.700	562.3	.750

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
562.3	.750	631.0	.800	707.9	.850	794.3	.900	891.3	.950
563.6	.751	632.4	.801	709.6	.851	796.2	.901	893.3	.951
564.9	.752	633.9	.802	711.2	.852	798.0	.902	895.4	.952
566.2	.753	635.3	.803	712.9	.853	799.8	.903	897.4	.953
567.5	.754	636.8	.804	714.5	.854	801.7	.904	899.5	.954
568.9	.755	638.3	.805	716.1	.855	803.5	.905	901.6	.955
570.2	.756	639.7	.806	717.8	.856	805.4	.906	903.6	.956
571.5	.757	641.2	.807	719.4	.857	807.2	.907	905.7	.957
572.8	.758	642.7	.808	721.1	.858	809.1	.908	907.8	.958
574.1	.759	644.2	.809	722.8	.859	811.0	.909	909.9	.959
575.4	.760	645.7	.810	724.4	.860	812.8	.910	912.0	.960
576.8	.761	647.1	.811	726.1	.861	814.7	.911	914.1	.961
578.1	.762	648.6	.812	727.8	.862	816.6	.912	916.2	.962
579.4	.763	650.1	.813	729.5	.863	818.5	.913	918.3	.963
580.8	.764	651.6	.814	731.1	.864	820.4	.914	920.4	.964
582.1	.765	653.1	.815	732.8	.865	822.2	.915	922.6	.965
583.4	.766	654.5	.816	734.5	.866	824.1	.916	924.7	.966
584.8	.767	656.1	.817	736.2	.867	826.0	.917	926.8	.967
586.1	.768	657.7	.818	737.9	.868	827.9	.918	929.0	.968
587.5	.769	659.2	.819	739.6	.869	829.9	.919	931.1	.969
588.8	.770	660.7	.820	741.3	.870	831.8	.920	933.3	.970
590.2	.771	662.2	.821	743.0	.871	833.7	.921	935.4	.971
591.6	.772	663.7	.822	744.7	.872	835.6	.922	937.6	.972
592.9	.773	665.3	.823	746.4	.873	837.5	.923	939.7	.973
594.3	.774	666.8	.824	748.2	.874	839.5	.924	941.9	.974
595.7	.775	668.3	.825	749.9	.875	841.4	.925	944.1	.975
597.0	.776	669.9	.826	751.6	.876	843.3	.926	946.2	.976
598.4	.777	671.4	.827	753.4	.877	845.3	.927	948.4	.977
599.8	.778	673.0	.828	755.1	.878	847.2	.928	950.6	.978
601.2	.779	674.5	.829	756.8	.879	849.2	.929	952.8	.979
602.6	.780	676.1	.830	758.6	.880	851.1	.930	955.0	.980
603.9	.781	677.6	.831	760.3	.881	853.1	.931	957.2	.981
605.3	.782	679.2	.832	762.1	.882	855.1	.932	959.4	.982
606.7	.783	680.8	.833	763.8	.883	857.0	.933	961.6	.983
608.1	.784	682.3	.834	765.6	.884	859.0	.934	963.8	.984
609.5	.785	683.9	.835	767.4	.885	861.0	.935	966.1	.985
610.9	.786	685.5	.836	769.1	.886	863.0	.936	968.3	.986
612.4	.787	687.1	.837	770.9	.887	865.0	.937	970.5	.987
613.8	.788	688.7	.838	772.7	.888	867.0	.938	972.7	.988
615.2	.789	690.2	.839	774.5	.889	869.0	.939	975.0	.989
616.6	.790	691.8	.840	776.2	.890	871.0	.940	977.2	.990
618.0	.791	693.4	.841	778.0	.891	873.0	.941	979.5	.991
619.4	.792	695.0	.842	779.8	.892	875.0	.942	981.7	.992
620.9	.793	696.6	.843	781.6	.893	877.0	.943	984.0	.993
622.3	.794	698.2	.844	783.4	.894	879.0	.944	986.3	.994
623.7	.795	699.8	.845	785.2	.895	881.0	.945	988.6	.995
625.2	.796	701.5	.846	787.0	.896	883.1	.946	990.8	.996
626.6	.797	703.1	.847	788.9	.897	885.1	.947	993.1	.997
628.1	.798	704.7	.848	790.7	.898	887.2	.948	995.4	.998
629.5	.799	706.3	.849	792.5	.899	889.2	.949	997.7	.999
631.0	.800	707.9	.850	794.3	.900	891.3	.950	1000	.000

N.	L.	0	1	2	3	4	5	6	7	8	9	Δ	P.P.	
0	—	0000	3010	4771	6021	6990	7782	8451	9031	9542				
1	.0000	0414	0792	1139	1461	1761	2041	2304	2553	2788				
2	.3010	3222	3424	3617	3802	3979	4150	4314	4472	4624				
3	.4771	4914	5051	5185	5315	5441	5563	5682	5798	5911				
4	.6021	6128	6232	6335	6435	6532	6628	6721	6812	6902			Δn	$\Delta 1$
5	.6990	7076	7160	7243	7324	7404	7482	7559	7634	7709			0.5	1
6	.7782	7853	7924	7993	8062	8129	8195	8261	8325	8388			0.9	2
7	.8451	8513	8573	8633	8692	8751	8808	8865	8921	8976			1.4	3
8	.9031	9085	9138	9191	9243	9294	9345	9395	9445	9494			1.8	4
9	.9542	9590	9638	9685	9731	9777	9823	9868	9912	9956			2.3	5
10	.0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	40	2.7	6	
11	.0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	37	3.2	7	
12	.0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	33	3.6	8	
13	.1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	31	4.1	9	
14	.1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	29	4.5	10	
15	.1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	27	5.0	11	
16	.2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	25	5.5	12	
17	.2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	24	5.9	13	
18	.2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	23	6.4	14	
19	.2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	21	6.8	15	
20	.3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	21	7.3	16	
21	.3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	20	7.7	17	
22	.3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	19	8.2	18	
23	.3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	18	8.6	19	
24	.3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	17	9.1	20	
25	.3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	17	9.5	21	
26	.4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	16	10.0	22	
27	.4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	16			
28	.4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	15			
29	.4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	14			
30	.4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	14			
31	.4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	13			
32	.5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	13	Δn	$\Delta 1$	
33	.5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	13	0.7	1	
34	.5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	13	1.3	2	
35	.5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	12	2.0	3	
36	.5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	12	2.7	4	
37	.5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	12	3.3	5	
38	.5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	12	4.0	6	
39	.5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	11	4.7	7	
40	.6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	11	5.3	8	
41	.6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	10	6.0	9	
42	.6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	10	6.7	10	
43	.6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	10	7.3	11	
44	.6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	10	8.0	12	
45	.6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	10	8.7	13	
46	.6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	9	9.3	14	
47	.6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	9	10.0	15	
48	.6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	9			
49	.6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	9			
50	.6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	9			

[illegible][illegible]

N.	L. o	1	2	3	4	5	6	7	8	9	P.P.
50	.6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	9 Δ 1
51	.7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	
52	.7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	
53	.7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	
54	.7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1·1 1
55	.7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	
56	.7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	
57	.7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	
58	.7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	2·2 2
59	.7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	
60	.7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	
61	.7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	
62	.7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	3·3 3
63	.7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	
64	.8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	
65	.8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	
66	.8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	4·4 4
67	.8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	
68	.8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	
69	.8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	
70	.8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	5·6 5
71	.8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	
72	.8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	
73	.8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	
74	.8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	6·7 6
75	.8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	
76	.8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	
77	.8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	
78	.8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	7·8 7
79	.8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	
80	.9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	
81	.9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	8 Δ 1
82	.9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	
83	.9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	
84	.9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	
85	.9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1·4 1
86	.9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	
87	.9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	
88	.9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	
89	.9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	2·9 2
90	.9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	
91	.9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	
92	.9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	
93	.9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	3·3 3
94	.9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	
95	.9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	
96	.9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	
97	.9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	4·3 4
98	.9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	
99	.9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	
100	.0000	0004	0009	0013	0017	0022	0026	0030	0035	0039	

P.P.		N.	L. o	1	2	3	4	5	6	7	8	9
10	Δn $\Delta 1$	100	.0000	0004	0009	0013	0017	0022	0026	0030	0035	0039
		101	.0043	0048	0052	0056	0060	0065	0069	0073	0077	0082
		102	.0086	0090	0095	0099	0103	0107	0111	0116	0120	0124
		103	.0128	0133	0137	0141	0145	0149	0154	0158	0162	0166
		104	.0170	0175	0179	0183	0187	0191	0195	0199	0204	0208
	5	105	.0212	0216	0220	0224	0228	0233	0237	0241	0245	0249
		106	.0253	0257	0261	0265	0269	0273	0278	0282	0286	0290
		107	.0294	0298	0302	0306	0310	0314	0318	0322	0326	0330
		108	.0334	0338	0342	0346	0350	0354	0358	0362	0366	0370
		109	.0374	0378	0382	0386	0390	0394	0398	0402	0406	0410
10	5	110	.0414	0418	0422	0426	0430	0434	0438	0441	0445	0449
		111	.0453	0457	0461	0465	0469	0473	0477	0481	0484	0488
		112	.0492	0496	0500	0504	0508	0512	0515	0519	0523	0527
		113	.0531	0535	0538	0542	0546	0550	0554	0558	0561	0565
		114	.0569	0573	0577	0580	0584	0588	0592	0596	0599	0603
	Δn $\Delta 1$	115	.0507	0611	0615	0618	0622	0626	0630	0633	0637	0641
		116	.0645	0648	0652	0656	0660	0663	0667	0671	0674	0678
		117	.0682	0686	0689	0693	0697	0700	0704	0708	0711	0715
		118	.0719	0722	0726	0730	0734	0737	0741	0745	0748	0752
		119	.0755	0759	0763	0766	0770	0774	0777	0781	0785	0788
10	4	120	.0792	0795	0799	0803	0806	0810	0813	0817	0821	0824
		121	.0828	0831	0835	0839	0842	0846	0849	0853	0856	0860
		122	.0864	0867	0871	0874	0878	0881	0885	0888	0892	0896
		123	.0899	0903	0906	0910	0913	0917	0920	0924	0927	0931
		124	.0934	0938	0941	0945	0948	0952	0955	0959	0962	0966
	3	125	.0969	0973	0976	0980	0983	0986	0990	0993	0997	1000
		126	.1004	1007	1011	1014	1017	1021	1024	1028	1031	1035
		127	.1038	1041	1045	1048	1052	1055	1059	1062	1065	1069
		128	.1072	1075	1079	1082	1086	1089	1092	1096	1099	1103
		129	.1106	1109	1113	1116	1119	1123	1126	1129	1133	1136
10	Δn $\Delta 1$	130	.1139	1143	1146	1149	1153	1156	1159	1163	1166	1169
		131	.1173	1176	1179	1183	1186	1189	1193	1196	1199	1202
		132	.1206	1209	1212	1216	1219	1222	1225	1229	1232	1235
		133	.1239	1242	1245	1248	1252	1255	1258	1261	1265	1268
		134	.1271	1274	1278	1281	1284	1287	1290	1294	1297	1300
	2	135	.1303	1307	1310	1313	1316	1319	1323	1326	1329	1332
		136	.1335	1339	1342	1345	1348	1351	1355	1358	1361	1364
		137	.1367	1370	1374	1377	1380	1383	1386	1389	1392	1396
		138	.1399	1402	1405	1408	1411	1414	1418	1421	1424	1427
		139	.1430	1433	1436	1440	1443	1446	1449	1452	1455	1458
10	Δn $\Delta 1$	140	.1461	1464	1467	1471	1474	1477	1480	1483	1486	1489
		141	.1492	1495	1498	1501	1504	1508	1511	1514	1517	1520
		142	.1523	1526	1529	1532	1535	1538	1541	1544	1547	1550
		143	.1553	1556	1559	1562	1565	1569	1572	1575	1578	1581
		144	.1584	1587	1590	1593	1596	1599	1602	1605	1608	1611
	1	145	.1614	1617	1620	1623	1626	1629	1632	1635	1638	1641
		146	.1644	1647	1649	1652	1655	1658	1661	1664	1667	1670
		147	.1673	1676	1679	1682	1685	1688	1691	1694	1697	1700
		148	.1703	1706	1708	1711	1714	1717	1720	1723	1726	1729
		149	.1732	1735	1738	1741	1744	1746	1749	1752	1755	1758
10	Δn $\Delta 1$	150	.1761	1764	1767	1770	1772	1775	1778	1781	1784	1787

FOUR-PLACE LOGARITHMS.

N.	L.	0	1	2	3	4	5	6	7	8	9	P.P.
150	.1761	1764	1767	1770	1772	1775	1778	1781	1784	1787		
151	.1790	1793	1796	1798	1801	1804	1807	1810	1813	1816		
152	.1818	1821	1824	1827	1830	1833	1836	1838	1841	1844		
153	.1847	1850	1853	1855	1858	1861	1864	1867	1870	1872		
154	.1875	1878	1881	1884	1886	1889	1892	1895	1898	1901		
155	.1903	1906	1909	1912	1915	1917	1920	1923	1926	1928		
156	.1931	1934	1937	1940	1942	1945	1948	1951	1953	1956		
157	.1959	1962	1965	1967	1970	1973	1976	1978	1981	1984		
158	.1987	1989	1992	1995	1998	2000	2003	2006	2009	2011		
159	.2014	2017	2019	2022	2025	2028	2030	2033	2036	2038		
160	.2041	2044	2047	2049	2052	2055	2057	2060	2063	2066	Δn	3
161	.2068	2071	2074	2076	2079	2082	2084	2087	2090	2092	3.3	Δl
162	.2095	2098	2101	2103	2106	2109	2111	2114	2117	2119	6.7	1
163	.2122	2125	2127	2130	2133	2135	2138	2140	2143	2146	10	2
164	.2148	2151	2154	2156	2159	2162	2164	2167	2170	2172		3
165	.2175	2177	2180	2183	2185	2188	2191	2193	2196	2198		
166	.2201	2204	2206	2209	2212	2214	2217	2219	2222	2225		
167	.2227	2230	2232	2235	2238	2240	2243	2245	2248	2251		
168	.2253	2256	2258	2261	2263	2266	2269	2271	2274	2276		
169	.2279	2281	2284	2287	2289	2292	2294	2297	2299	2302		
170	.2304	2307	2310	2312	2315	2317	2320	2322	2325	2327		
171	.2330	2333	2335	2338	2340	2343	2345	2348	2350	2353		
172	.2355	2358	2360	2363	2365	2368	2370	2373	2375	2378		
173	.2380	2383	2385	2388	2390	2393	2395	2398	2400	2403		
174	.2405	2408	2410	2413	2415	2418	2420	2423	2425	2428		
175	.2430	2433	2435	2438	2440	2443	2445	2448	2450	2453	Δn	2
176	.2455	2458	2460	2463	2465	2467	2470	2472	2475	2477	5.0	Δl
177	.2480	2482	2485	2487	2490	2492	2494	2497	2499	2502	10	1
178	.2504	2507	2509	2512	2514	2516	2519	2521	2524	2526		2
179	.2529	2531	2533	2536	2538	2541	2543	2545	2548	2550		
180	.2553	2555	2558	2560	2562	2565	2567	2570	2572	2574		
181	.2577	2579	2582	2584	2586	2589	2591	2594	2596	2598		
182	.2601	2603	2605	2608	2610	2613	2615	2617	2620	2622		
183	.2625	2627	2629	2632	2634	2636	2639	2641	2643	2646		
184	.2648	2651	2653	2655	2658	2660	2662	2665	2667	2669		
185	.2672	2674	2676	2679	2681	2683	2686	2688	2690	2693		
186	.2695	2697	2700	2702	2704	2707	2709	2711	2714	2716		
187	.2718	2721	2723	2725	2728	2730	2732	2735	2737	2739		
188	.2742	2744	2746	2749	2751	2753	2755	2758	2760	2762		
189	.2765	2767	2769	2772	2774	2776	2778	2781	2783	2785	Δn	1
190	.2788	2790	2792	2794	2797	2799	2801	2804	2806	2808	10	Δl
191	.2810	2813	2815	2817	2819	2822	2824	2826	2828	2831		1
192	.2833	2835	2838	2840	2842	2844	2847	2849	2851	2853		
193	.2856	2858	2860	2862	2865	2867	2869	2871	2874	2876		
194	.2878	2880	2882	2885	2887	2889	2891	2894	2896	2898		
195	.2900	2903	2905	2907	2909	2911	2914	2916	2918	2920		
196	.2923	2925	2927	2929	2931	2934	2936	2938	2940	2942		
197	.2945	2947	2949	2951	2953	2956	2958	2960	2962	2964		
198	.2967	2969	2971	2973	2975	2978	2980	2982	2984	2986		
199	.2989	2991	2993	2995	2997	2999	3002	3004	3006	3008		
200	.3010	3012	3015	3017	3019	3021	3023	3025	3028	3030		

TABLE FOR THE CALCULATION OF CHEMICAL FORMULÆ.

Taking x and y as symbols for integers less than 31, the table gives directly the decimal values F of all ratios $x : y$ up to $x : y = 0.5$. For the *approximate* reduction of a given value F_0 to an integer-ratio, if $F_0 < 0.5$, find (if possible) that tabular F which does not differ from F_0 by more than you can (or must) tolerate. If $F_0 > 0.5$, but less than 1, operate upon $1 - F_0$, and subtract the corresponding $x' : y'$ from $y' : y'$. If $F_0 = a + f_0$ where $a = 1, 2, 3 \dots$; find the $x'' : y''$ corresponding to f_0 and say: $a + f_0 = a + (x'' : y'') = (ay'' + x'') : y''$. Or else operate upon the reciprocal $1/F_0$.

A table going up to $y = 60$ is given on pp. 48 *et seqq.*

F.	$x : y$.	F.	$x : y$.	F.	$x : y$.	F.	$x : y$.
.03333	1 : 30	.1333	2 : 15	.2609	6 : 23	.3810	8 : 21
.03448	1 : 29	.1364	3 : 22	.2632	5 : 19	.3846	5 : 13
.03571	1 : 28	.1379	4 : 29	.2667	4 : 15	.3889	7 : 18
.03704	1 : 27	.1429	1 : 7	.2692	7 : 26	.3913	9 : 23
.03846	1 : 26	.1481	4 : 27	.2727	3 : 11	.3929	11 : 28
.04000	1 : 25	.1500	3 : 20	.2759	8 : 29	.4000	2 : 5
.04167	1 : 24	.1539	2 : 13	.2778	5 : 18	.4074	11 : 27
.04348	1 : 23	.1579	3 : 19	.2800	7 : 25	.4091	9 : 22
.04546	1 : 22	.1600	4 : 25	.2857	2 : 7	.4118	7 : 17
.04762	1 : 21	.1667	1 : 6	.2917	7 : 24	.4138	12 : 29
.05000	1 : 20	.1724	5 : 29	.2941	5 : 17	.4166	5 : 12
.05263	1 : 19	.1739	4 : 23	.2963	8 : 27	.4211	8 : 19
.05556	1 : 18	.1765	3 : 17	.3000	3 : 10	.4231	11 : 26
.05882	1 : 17	.1786	5 : 28	.3043	7 : 23	.4285	3 : 7
.06250	1 : 16	.1818	2 : 11	.3077	4 : 13	.4333	13 : 30
.06667	1 : 15	.1852	5 : 27	.3103	9 : 29	.4348	10 : 23
.06897	2 : 29	.1875	3 : 16	.3125	5 : 16	.4375	7 : 16
.07143	1 : 14	.1905	4 : 21	.3158	6 : 19	.4400	11 : 25
.07407	2 : 27	.1923	5 : 26	.3182	7 : 22	.4444	4 : 9
.07692	1 : 13	.2000	1 : 5	.3200	8 : 25	.4483	13 : 29
.08000	2 : 25	.2069	6 : 29	.3214	9 : 28	.4500	9 : 20
.08333	1 : 12	.2083	5 : 24	.3333	1 : 3	.4545	5 : 11
.08696	2 : 23	.2105	4 : 19	.3448	10 : 29	.4583	11 : 24
.09091	1 : 11	.2143	3 : 14	.3462	9 : 26	.4615	6 : 13
.09524	2 : 21	.2174	5 : 23	.3478	8 : 23	.4643	13 : 28
.1000	1 : 10	.2222	2 : 9	.3500	7 : 20	.4667	7 : 15
.1034	3 : 29	.2273	5 : 22	.3529	6 : 17	.4706	8 : 17
.1053	2 : 19	.2308	3 : 13	.3571	5 : 14	.4737	9 : 19
.1071	3 : 28	.2333	7 : 30	.3600	9 : 25	.4762	10 : 21
.1111	1 : 9	.2353	4 : 17	.3636	4 : 11	.4783	11 : 23
.1154	3 : 26	.2381	5 : 21	.3667	11 : 30	.4800	12 : 25
.1176	2 : 17	.2400	6 : 25	.3684	7 : 19	.4815	13 : 27
.1200	3 : 25	.2414	7 : 29	.3704	10 : 27	.4828	14 : 29
.1250	1 : 8	.2500	1 : 4	.3750	3 : 8	.5000	1 : 2
.1304	3 : 23	.2593	7 : 27	.3793	11 : 29		

n.	$\frac{1}{n}$	P.P.	n.	$\frac{1}{n}$	P.P.	n.	$\frac{1}{n}$	P.P.
1000	10000		1100	9091		1200	8333	
1002	9980		1102	9074		1202	8319	
1004	9960		1104	9058		1204	8306	Δn $\Delta \frac{1}{n}$
1006	9940		1106	9042		1206	8292	
1008	9921	Δn $\Delta \frac{1}{n}$	1108	9025	Δn $\Delta \frac{1}{n}$	1208	8278	
1010	9901	·2 2	1110	9009	·2 2	1210	8264	·1 1
1012	9881	·4 4	1112	8993	·5 4	1212	8251	·3 2
1014	9862	·6 6	1114	8977	·7 6	1214	8237	·4 3
1016	9843	·8 8	1116	8961	1·0 8	1216	8224	·6 4
1018	9823	1·0 10	1118	8945	1·2 10	1218	8210	·7 5
		1·2 12			1·5 12			·9 6
1020	9804	1·4 14	1120	8929	1·7 14	1220	8197	1·0 7
1022	9785	1·7 16	1122	8913	2·0 16	1222	8183	1·2 8
1024	9766	1·9 18	1124	8897	2·2 18	1224	8170	1·3 9
1026	9747	2·1 20	1126	8881	2·5 20	1226	8157	1·5 10
1028	9728		1128	8865		1228	8143	1·6 11
								1·8 12
1030	9709		1130	8850		1230	8130	1·9 13
1032	9690		1132	8834		1232	8117	2·1 14
1034	9671		1134	8818		1234	8104	
1036	9653		1136	8803		1236	8091	
1038	9634		1138	8787		1238	8078	
								Δn $\Delta \frac{1}{n}$
1040	9615	Δn $\Delta \frac{1}{n}$	1140	8772	Δn $\Delta \frac{1}{n}$	1240	8065	·2 1
1042	9597		1142	8757		1242	8052	·3 2
1044	9579	·2 2	1144	8741	·3 2	1244	8039	·5 3
1046	9560	·4 4	1146	8726	·5 4	1246	8026	·6 4
1048	9542	·7 6	1148	8711	·8 6	1248	8013	·8 5
		·9 8			1·1 8			·9 6
1050	9524	1·1 10	1150	8696	1·3 10	1250	8000	1·1 7
1052	9506	1·3 12	1152	8681	1·6 12	1252	7987	1·3 8
1054	9488	1·5 14	1154	8666	1·9 14	1254	7974	1·4 9
1056	9470	1·8 16	1156	8651	2·1 16	1256	7962	1·6 10
1058	9452	2·0 18	1158	8636	2·4 18	1258	7949	1·7 11
		2·2 20			2·7 20			1·9 12
1060	9434		1160	8621		1260	7937	2·0 13
1062	9416		1162	8606		1262	7924	
1064	9398		1164	8591		1264	7911	
1066	9381		1166	8576		1266	7899	
1068	9363		1168	8562		1268	7886	
								Δn $\Delta \frac{1}{n}$
1070	9346		1170	8547		1270	7874	
1072	9328		1172	8532		1272	7862	
1074	9311	Δn $\Delta \frac{1}{n}$	1174	8518	Δn $\Delta \frac{1}{n}$	1274	7849	·2 1
1076	9294		1176	8503	·3 2	1276	7837	·3 2
1078	9276	·2 2	1178	8489	·6 4	1278	7825	·5 3
		·5 4			·8 6			·7 4
1080	9259	·7 6	1180	8475	1·1 8	1280	7813	·8 5
1082	9242	·9 8	1182	8460	1·4 10	1282	7800	1·0 6
1084	9225	1·2 10	1184	8446	1·7 12	1284	7788	1·2 7
1086	9208	1·4 12	1186	8432	2·0 14	1286	7776	1·3 8
1088	9191	1·6 14	1188	8418	2·2 16	1288	7764	1·5 9
		1·9 16			2·5 18			1·6 10
1090	9174	2·1 18	1190	8403	2·8 20	1290	7752	1·8 11
1092	9158	2·4 20	1192	8389		1292	7740	2·0 12
1094	9141		1194	8375		1294	7728	2·1 13
1096	9124		1196	8361		1296	7716	
1098	9107		1198	8347		1298	7704	
1100	9091		1200	8333		1300	7692	

n.	$\frac{1}{n}$	P.P.	n.	$\frac{1}{n}$	P.P.	n.	$\frac{1}{n}$	P.P.
1300	7692		1400	7143		1514	6605	Δn $\Delta \frac{1}{n}$
1302	7680		1402	7133		1517	6592	
1304	7669		1404	7123		1521	6575	-2 1
1306	7657	Δn $\Delta \frac{1}{n}$	1406	7112	Δn $\Delta \frac{1}{n}$	1524	6562	-5 2
1308	7645		1408	7102		1528	6545	-7 3
		-2 1			-2 1			1.0 4
1310	7634	-3 2	1410	7092	-4 2	1531	6532	1.2 5
1312	7622	-5 3	1412	7082	-6 3	1535	6515	1.4 6
1314	7610	-7 4	1414	7072	-8 4	1538	6502	1.7 7
1316	7599	-9 5	1416	7062	1.0 5	1542	6485	1.9 8
1318	7587	1.0 6	1418	7052	1.2 6	1545	6472	2.1 9
		1.2 7			1.4 7			2.4 10
1320	7576	1.4 8	1420	7042	1.6 8	1549	6456	2.6 11
1322	7564	1.6 9	1422	7032	1.8 9	1552	6443	2.9 12
1324	7553	1.7 10	1424	7022	2.0 10	1556	6427	3.1 13
1326	7541	1.9 11	1426	7013	2.2 11	1560	6410	3.3 14
1328	7530	2.1 12	1428	7003		1563	6398	3.6 15
								3.8 16
1330	7519		1430	6993		1567	6382	4.1 17
1332	7508		1432	6983		1570	6369	
1334	7496		1434	6974		1574	6353	
1336	7485		1436	6964		1578	6337	Δn $\Delta \frac{1}{n}$
1338	7474		1438	6954		1581	6325	
		Δn $\Delta \frac{1}{n}$			Δn $\Delta \frac{1}{n}$			-3 1
1340	7463	-2 1	1440	6944	-2 1	1585	6309	-5 2
1342	7452	-4 2	1442	6935	-4 2	1589	6293	-8 3
1344	7440	-5 3	1444	6925	-6 3	1592	6281	1.0 4
1346	7429	-7 4	1446	6916	-8 4	1596	6266	1.3 5
1348	7418	-9 5	1448	6906	-1.1 5	1600	6250	1.5 6
		1.1 6			1.3 6			1.8 7
1350	7407	1.3 7	1450	6897	1.5 7	1603	6238	2.1 8
1352	7396	1.5 8	1452	6887	1.7 8	1607	6223	2.3 9
1354	7386	1.6 9	1454	6878	1.9 9	1611	6207	2.6 10
1356	7375	1.8 10	1456	6868	2.1 10	1614	6196	2.8 11
1358	7364	2.0 11	1458	6859		1618	6180	3.1 12
		2.2 12						3.3 13
1360	7353		1460	6849		1622	6165	3.6 14
1362	7342		1462	6840		1626	6150	3.9 15
1364	7331		1464	6831		1629	6139	4.1 16
1366	7321		1466	6821		1633	6124	
1368	7310		1468	6812		1637	6109	
					Δn $\Delta \frac{1}{n}$			Δn $\Delta \frac{1}{n}$
1370	7299		1470	6803	-2 1	1641	6094	-3 1
1372	7289		1472	6793	-4 2	1644	6083	-6 2
1374	7278	Δn $\Delta \frac{1}{n}$	1474	6784	-7 3	1648	6068	-8 3
1376	7267	-2 1	1476	6775	-9 4	1652	6053	1.1 4
1378	7257	-4 2	1478	6766	-1.1 5	1656	6039	1.4 5
		-6 3			1.3 6			1.7 6
1380	7246	-8 4	1480	6757	1.6 7	1660	6024	1.9 7
1382	7236	-1.0 5	1483	6743	1.8 8	1663	6013	2.2 8
1384	7225	1.1 6	1486	6729	2.0 9	1667	5999	2.5 9
1386	7215	1.3 7	1489	6716	2.2 10	1671	5984	2.8 10
1388	7205	1.5 8	1493	6698	2.4 11	1675	5970	3.1 11
		1.7 9			2.7 12			3.3 12
1390	7194	1.9 10	1496	6684	2.9 13	1679	5956	3.6 13
1392	7184	2.1 11	1500	6667	3.1 14	1683	5942	3.9 14
1394	7174		1503	6653	3.3 15	1687	5928	4.2 15
1396	7163		1507	6636	3.6 16	1690	5917	
1398	7153		1510	6623	3.8 17	1694	5903	
					4.0 18			
1400	7143		1514	6605		1698	5889	

n.	$\frac{1}{n}$	P.P.	n.	$\frac{1}{n}$	P.P.	n.	$\frac{1}{n}$	P.P.
1700	5882		1905	5249		2155	4640	
1702	5875		1910	5236		2160	4630	
1706	5862	Δn	1915	5222	$\Delta \frac{1}{n}$	2165	4619	Δn
1710	5848	$\Delta \frac{1}{n}$	1920	5208	Δn	2170	4608	$\Delta \frac{1}{n}$
1714	5834	$\cdot 3$	1925	5195	$\cdot 4$	2175	4598	$\cdot 5$
		$\cdot 6$			$\cdot 8$			$\cdot 10$
1718	5821	$\cdot 9$	1930	5181	$\cdot 1$	2180	4587	$\cdot 1$
1722	5807	$\cdot 2$	1935	5168	$\cdot 5$	2185	4577	$\cdot 4$
1726	5794	$\cdot 5$	1940	5155	$\cdot 9$	2190	4566	$\cdot 8$
1730	5780	$\cdot 8$	1945	5141	$\cdot 3$	2195	4556	$\cdot 5$
1734	5767	$\cdot 1$	1950	5128	$\cdot 7$	2200	4545	$\cdot 9$
		$\cdot 4$			$\cdot 0$			$\cdot 3$
1738	5754	$\cdot 7$	1955	5115	$\cdot 4$	2205	4535	$\cdot 9$
1742	5741	$\cdot 0$	1960	5102	$\cdot 8$	2210	4525	$\cdot 4$
1746	5727	$\cdot 3$	1965	5089	$\cdot 2$	2215	4515	$\cdot 8$
1750	5714	$\cdot 6$	1970	5076	$\cdot 5$	2220	4505	$\cdot 3$
1754	5701	$\cdot 9$	1975	5063	$\cdot 9$	2225	4494	$\cdot 8$
		$\cdot 13$			$\cdot 13$			$\cdot 12$
1758	5688		1980	5051		2230	4484	
1762	5675		1985	5038		2235	4474	
1766	5663		1990	5025		2240	4464	
1770	5650	Δn	1995	5013	$\Delta \frac{1}{n}$	2245	4454	Δn
1774	5637	$\Delta \frac{1}{n}$	2000	5000	$\Delta \frac{1}{n}$	2250	4444	$\Delta \frac{1}{n}$
		$\cdot 3$	2005	4988	$\cdot 4$	2255	4435	$\cdot 5$
1778	5624	$\cdot 6$	2010	4975	$\cdot 8$	2260	4425	$\cdot 10$
1782	5612	$\cdot 0$	2015	4963	$\cdot 2$	2265	4415	$\cdot 5$
1786	5599	$\cdot 3$	2020	4950	$\cdot 6$	2270	4405	$\cdot 10$
1791	5583	$\cdot 6$	2025	4938	$\cdot 1$	2275	4396	$\cdot 5$
1795	5571	$\cdot 9$			$\cdot 5$			$\cdot 10$
		$\cdot 3$	2030	4926	$\cdot 9$	2280	4386	$\cdot 5$
1799	5559	$\cdot 6$	2035	4914	$\cdot 3$	2285	4376	$\cdot 10$
1803	5546	$\cdot 0$	2040	4902	$\cdot 7$	2290	4367	$\cdot 5$
1807	5534	$\cdot 3$	2045	4890	$\cdot 1$	2295	4357	$\cdot 10$
1811	5522	$\cdot 6$	2050	4878	$\cdot 5$	2300	4348	$\cdot 5$
1816	5507	$\cdot 9$			$\cdot 0$			$\cdot 10$
		$\cdot 13$	2055	4866	$\cdot 4$	2305	4338	
1820	5495	$\cdot 4$	2060	4854	$\cdot 8$	2310	4329	
1824	5482	$\cdot 7$	2065	4843	$\cdot 2$	2315	4320	
1828	5470	$\cdot 1$	2070	4831	$\cdot 6$	2320	4310	
1832	5459	$\cdot 4$	2075	4819	$\cdot 0$	2325	4301	
1837	5444							
1841	5432	Δn	2080	4808	$\Delta \frac{1}{n}$	2330	4292	Δn
1845	5420	$\Delta \frac{1}{n}$	2085	4796	$\Delta \frac{1}{n}$	2335	4283	$\Delta \frac{1}{n}$
1849	5408	$\cdot 3$	2090	4785	$\cdot 4$	2340	4274	$\cdot 6$
1854	5394	$\cdot 7$	2095	4773	$\cdot 9$	2345	4264	$\cdot 1$
1858	5382	$\cdot 1$	2100	4762	$\cdot 3$	2350	4255	$\cdot 5$
		$\cdot 4$			$\cdot 8$			$\cdot 10$
1862	5371	$\cdot 7$	2105	4751	$\cdot 1$	2355	4246	$\cdot 5$
1866	5359	$\cdot 1$	2110	4739	$\cdot 5$	2360	4237	$\cdot 10$
1871	5345	$\cdot 4$	2115	4728	$\cdot 9$	2365	4228	
1875	5333	$\cdot 8$	2120	4717	$\cdot 3$	2370	4219	
1879	5322	$\cdot 1$	2125	4706	$\cdot 7$	2375	4211	
		$\cdot 5$			$\cdot 1$			
1884	5308	$\cdot 8$	2130	4695	$\cdot 5$	2380	4202	
1888	5297	$\cdot 2$	2135	4684	$\cdot 9$	2385	4193	
1892	5285	$\cdot 6$	2140	4673	$\cdot 3$	2390	4184	
1897	5271	$\cdot 0$	2145	4662	$\cdot 7$	2395	4175	
1901	5260	$\cdot 4$	2150	4651	$\cdot 1$	2400	4167	
		$\cdot 8$			$\cdot 5$			
1905	5249		2155	4640		2405	4158	

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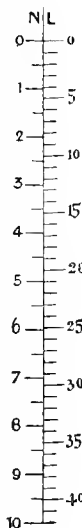
n.	$\frac{1}{n}$	P.P.	n.	$\frac{1}{n}$	P.P.	n.	$\frac{1}{n}$	P.P.
2405	4158		2655	3766		2905	3442	
2410	4149		2660	3759		2910	3436	
2415	4141		2665	3752		2915	3431	
2420	4132		2670	3745		2920	3425	
2425	4124	Δn $\Delta \frac{1}{n}$	2675	3738	Δn $\Delta \frac{1}{n}$	2925	3419	Δn $\Delta \frac{1}{n}$
2430	4115	$\cdot 6$ 1	2680	3731	$\cdot 7$ 1	2930	3413	$\cdot 9$ 1
2435	4107	1·2 2	2685	3724	1·5 2	2935	3407	1·7 2
2440	4098	1·8 3	2690	3717	2·2 3	2940	3401	2·6 3
2445	4090	2·4 4	2695	3711	2·9 4	2945	3396	3·5 4
2450	4082	3·0 5	2700	3704	3·6 5	2950	3390	4·3 5
		3·6 6			4·4 6			5·2 6
2455	4073	4·2 7	2705	3697	5·1 7	2955	3384	6·1 7
2460	4065	4·8 8	2710	3690		2960	3378	
2465	4057	5·4 9	2715	3683		2965	3373	
2470	4049	6·0 10	2720	3676		2970	3367	
2475	4040		2725	3670		2975	3361	
2480	4032		2730	3663		2980	3356	
2485	4024		2735	3656		2985	3350	
2490	4016		2740	3650		2990	3344	
2495	4008		2745	3643		2995	3339	
2500	4000		2750	3636		3000	3333	
2505	3992		2755	3630		3005	3328	
2510	3984	Δn $\Delta \frac{1}{n}$	2760	3623	Δn $\Delta \frac{1}{n}$	3010	3322	Δn $\Delta \frac{1}{n}$
2515	3976	$\cdot 6$ 1	2765	3617	$\cdot 8$ 1	3015	3317	$\cdot 9$ 1
2520	3968	1·3 2	2770	3610	1·5 2	3020	3311	1·8 2
2525	3960	1·9 3	2775	3604	2·3 3	3025	3306	2·8 3
2530	3953	2·6 4	2780	3597	3·1 4	3030	3300	3·7 4
2535	3945	3·2 5	2785	3591	3·9 5	3035	3295	4·6 5
2540	3937	3·8 6	2790	3584	4·6 6	3040	3289	5·5 6
2545	3929	4·5 7	2795	3578	5·4 7	3045	3284	
2550	3922	5·1 8	2800	3571		3050	3279	
2555	3914		2805	3565		3055	3273	
2560	3906		2810	3559		3060	3268	
2565	3899		2815	3552		3065	3263	
2570	3891		2820	3546		3070	3257	
2575	3883		2825	3540		3075	3252	
2580	3876		2830	3534		3080	3247	
2585	3868		2835	3527		3085	3241	Δn $\Delta \frac{1}{n}$
2590	3861		2840	3521		3090	3236	$\cdot 9$ 1
2595	3854	Δn $\Delta \frac{1}{n}$	2845	3515	Δn $\Delta \frac{1}{n}$	3095	3231	1·0 1
2600	3846	$\cdot 7$ 1	2850	3509	1·6 2	3100	3226	1·9 2
2605	3839	1·4 2	2855	3503	2·5 3	3105	3221	2·9 3
2610	3831	2·1 3	2860	3497	3·3 4	3110	3215	3·9 4
2615	3824	2·7 4	2865	3490	4·1 5	3115	3210	4·9 5
2620	3817	3·4 5	2870	3484	4·9 6	3120	3205	5·8 6
2625	3810	4·1 6	2875	3478	5·7 7	3125	3200	6·8 7
		4·8 7						7·8 8
2630	3802	5·5 8	2880	3472		3130	3195	8·8 9
2635	3795		2885	3466		3135	3190	9·7 10
2640	3788		2890	3460		3140	3185	10·7 11
2645	3781		2895	3454		3145	3175	11·7 12
2650	3774		2900	3448		3160	3165	
2655	3766		2905	3442		3162·3	3162·3	

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
100	00	000	043	087	130	173	217	260	303	346	389	43	44
101		432	475	518	561	604	647	689	732	775	817	43	
102		860	903	945	988	$\downarrow$ 30	$\downarrow$ 72	115	157	199	242	42	
103	01	284	326	368	410	452	494	536	578	620	662	41	
104		703	745	787	828	870	912	953	995	$\downarrow$ 36	$\downarrow$ 78	41	N L
105	02	119	160	202	243	284	325	366	407	449	490	41	
106		531	572	612	653	694	735	776	816	857	898	40	
107		938	979	$\downarrow$ 19	$\downarrow$ 60	100	141	181	222	262	302	40	
108	03	342	383	423	463	503	543	583	623	663	703	40	
109		743	782	822	862	902	941	981	$\downarrow$ 21	$\downarrow$ 60	100	39	
110	04	139	179	218	258	297	336	376	415	454	493	39	
111		532	571	610	650	689	727	766	805	844	883	39	
112		922	961	999	$\downarrow$ 38	$\downarrow$ 77	115	154	192	231	269	39	
113	05	308	346	385	423	461	500	538	576	614	652	38	
114		690	729	767	805	843	881	918	956	994	$\downarrow$ 32	38	
115	06	070	108	145	183	221	258	296	333	371	408	38	N L
116		446	483	521	558	595	633	670	707	744	781	38	
117		819	856	893	930	967	$\downarrow$ 04	$\downarrow$ 41	$\downarrow$ 78	115	151	37	
118	07	188	225	262	298	335	372	408	445	482	518	37	
119		555	591	628	664	700	737	773	809	846	882	36	
120		918	954	990	$\downarrow$ 27	$\downarrow$ 63	$\downarrow$ 99	135	171	207	243	36	
121	08	279	314	350	386	422	458	493	529	565	600	36	
122		636	672	707	743	778	814	849	884	920	955	36	
123		991	$\downarrow$ 26	$\downarrow$ 61	$\downarrow$ 96	132	167	202	237	272	307	35	
124	09	342	377	412	447	482	517	552	587	621	656	35	
125		691	726	760	795	830	864	899	934	968	$\downarrow$ 03	34	N L
126	10	037	072	106	140	175	209	243	278	312	346	34	
127		380	415	449	483	517	551	585	619	653	687	34	
128		721	755	789	823	857	890	924	958	992	$\downarrow$ 25	34	
129	11	059	093	126	160	193	227	261	294	327	361	33	
130		394	428	461	494	528	561	594	628	661	694	33	
131		727	760	793	826	860	893	926	959	992	$\downarrow$ 24	33	
132	12	057	090	123	156	189	222	254	287	320	352	33	
133		385	418	450	483	516	548	581	613	646	678	32	
134		710	743	775	808	840	872	905	937	969	$\downarrow$ 01	32	
135	13	033	066	098	130	162	194	226	258	290	322	32	N L
136		354	386	418	450	481	513	545	577	609	640	32	
137		672	704	735	767	799	830	862	893	925	956	32	
138		988	$\downarrow$ 19	$\downarrow$ 51	$\downarrow$ 82	114	145	176	208	239	270	31	
139	14	301	333	364	395	426	457	489	520	551	582	31	
140		613	644	675	706	737	768	799	829	860	891	31	
141		922	953	983	$\downarrow$ 14	$\downarrow$ 45	$\downarrow$ 76	106	137	168	198	31	
142	15	229	259	290	320	351	381	412	442	473	503	31	
143		534	564	594	625	655	685	715	746	776	806	30	
144		836	866	897	927	957	987	$\downarrow$ 17	$\downarrow$ 47	$\downarrow$ 77	107	30	
145	16	137	167	197	227	256	286	316	346	376	406	29	N L
146		435	465	495	524	554	584	613	643	673	702	30	
147		732	761	791	820	850	879	909	938	967	997	29	
148	17	026	056	085	114	143	173	202	231	260	289	30	
149		319	348	377	406	435	464	493	522	551	580	29	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

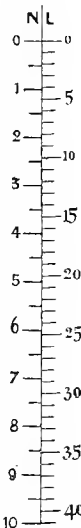
43



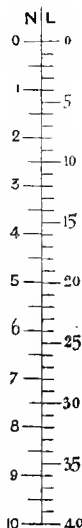
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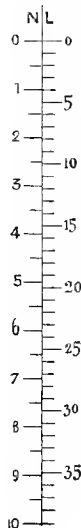
41



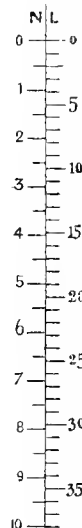
40



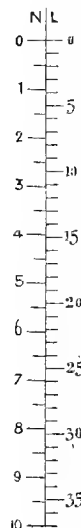
39



38



37



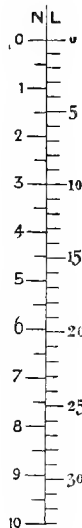
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34



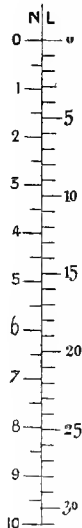
33



32



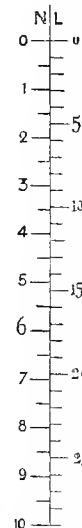
31



30



29



FIVE-PLACE LOGARITHMS.

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
150	17	609	638	667	696	725	754	782	811	840	869	29	
151		898	926	955	984	♦13	♦41	♦70	♦99	127	156	28	
152	18	184	213	241	270	298	327	355	384	412	441	28	
153		469	498	526	554	583	611	639	667	696	724	28	
154		752	780	808	837	865	893	921	949	977	♦05	28	
155	19	033	061	089	117	145	173	201	229	257	285	27	
156		312	340	368	396	424	451	479	507	535	562	28	
157		590	618	645	673	700	728	756	783	811	838	28	
158		866	893	921	948	976	♦03	♦30	♦58	♦85	112	28	
159	20	140	167	194	222	249	276	303	330	358	385	27	
160		412	439	466	493	520	548	575	602	629	656	27	
161		683	710	737	763	790	817	844	871	898	925	27	
162		952	978	♦05	♦32	♦59	♦85	112	139	165	192	27	
163	21	219	245	272	299	325	352	378	405	431	458	26	
164		484	511	537	564	590	617	643	669	696	722	26	
165		748	775	801	827	854	880	906	932	958	985	26	
166	22	011	037	063	089	115	141	167	194	220	246	26	
167		272	298	324	350	376	401	427	453	479	505	26	
168		531	557	583	608	634	660	686	712	737	763	26	
169		789	814	840	866	891	917	943	968	994	♦19	26	
170	23	045	070	096	121	147	172	198	223	249	274	26	
171		300	325	350	376	401	426	452	477	502	528	25	
172		553	578	603	629	654	679	704	729	754	779	26	
173		805	830	855	880	905	930	955	980	♦05	♦30	25	
174	24	055	080	105	130	155	180	204	229	254	279	25	
175		304	329	353	378	403	428	452	477	502	527	24	
176		551	576	601	625	650	674	699	724	748	773	24	
177		797	822	846	871	895	920	944	969	993	♦18	24	
178	25	042	066	091	115	139	164	188	212	237	261	24	
179		285	310	334	358	382	406	431	455	479	503	24	
180		527	551	575	600	624	648	672	696	720	744	24	
181		768	792	816	840	864	888	912	935	959	983	24	
182	26	007	031	055	079	102	126	150	174	198	221	24	
183		245	269	293	316	340	364	387	411	435	458	24	
184		482	505	529	553	576	600	623	647	670	694	23	
185		717	741	764	788	811	834	858	881	905	928	23	
186		951	975	998	♦21	♦45	♦68	♦91	114	138	161	23	
187	27	184	207	231	254	277	300	323	346	370	393	23	
188		416	439	462	485	508	531	554	577	600	623	23	
189		646	669	692	715	738	761	784	807	830	852	23	
190		875	898	921	944	967	989	♦12	♦35	♦58	♦81	22	
191	28	103	126	149	171	194	217	240	262	285	307	23	
192		330	353	375	398	421	443	466	488	511	533	23	
193		556	578	601	623	646	668	691	713	735	758	22	
194		780	803	825	847	870	892	914	937	959	981	22	
195	29	003	026	048	070	092	115	137	159	181	203	23	
196		226	248	270	292	314	336	358	380	403	425	22	
197		447	469	491	513	535	557	579	601	623	645	22	
198		667	688	710	732	754	776	798	820	842	863	22	
199		885	907	929	951	973	994	♦16	♦38	♦60	♦81	22	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

29
Δ n Δ l

3 1
7 2
10 3
14 4
17 5
21 6
24 7
28 8
31 9
34 10
38 11
41 12
45 13
48 14
52 15
55 16
59 17
62 18
66 19
69 20
72 21
76 22
79 23
83 24
86 25
90 26
93 27
97 28
10 29

28		27		26		25	
Δn	Δl	Δn	Δl	Δn	Δl	Δn	Δl
4	1	4	1	4	1	4	1
7	2	7	2	8	2	8	2
11	3	11	3	12	3	12	3
14	4	15	4	16	4	16	4
18	5	19	5	19	5	20	5
21	6	22	6	23	6	24	6
25	7	26	7	27	7	28	7
29	8	30	8	31	8	32	8
32	9	33	9	35	9	36	9
36	10	37	10	38	10	40	10
39	11	41	11	42	11	44	11
43	12	44	12	46	12	48	12
46	13	48	13	50	13	52	13
50	14	62	14	54	14	56	14
54	15	56	15	58	15	60	15
57	16	69	16	62	16	64	16
61	17	63	17	65	17	68	17
64	18	67	18	69	18	72	18
68	19	70	19	73	19	76	19
71	20	74	20	77	20	80	20
75	21	78	21	81	21	84	21
79	22	81	22	85	22	88	22
82	23	85	23	88	23	92	23
86	24	89	24	92	24	96	24
89	25	93	25	96	25	10	25
93	26	96	26	10	26		
96	27	10	27				
10	28						

24		23		22		21	
Δn	Δl	Δn	Δl	Δn	Δl	Δn	Δl
4	1	4	1	5	1	5	1
8	2	9	2	9	2	10	2
13	3	13	3	14	3	14	3
17	4	17	4	18	4	19	4
21	5	22	5	23	5	24	5
25	6	26	6	27	6	29	6
29	7	30	7	32	7	33	7
33	8	35	8	36	8	38	8
38	9	39	9	41	9	43	9
42	10	43	10	45	10	48	10
46	11	48	11	50	11	52	11
50	12	52	12	55	12	57	12
54	13	57	13	60	13	62	13
58	14	61	14	64	14	67	14
63	15	65	15	68	15	71	15
67	16	70	16	73	16	76	16
71	17	74	17	77	17	81	17
75	18	78	18	82	18	86	18
79	19	83	19	86	19	90	19
83	20	87	20	91	20	95	20
88	21	91	21	95	21	10	21
92	22	96	22	10	22		
96	23	10	23				
10	24						

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
200	30	103	125	146	168	190	211	233	255	276	298	22	
201		320	341	363	384	406	428	449	471	492	514	21	
202		535	557	578	600	621	643	664	685	707	728	22	
203		750	771	792	814	835	856	878	899	920	942	21	
204		963	984	1006	1027	1048	1069	1091	1112	1133	1154	21	
205	31	175	197	218	239	260	281	302	323	345	366	21	22 21
206		387	408	429	450	471	492	513	534	555	576	21	Δ n Δ l Δ n Δ l
207		597	618	639	660	681	702	723	744	765	785	21	5 1 5 1
208		806	827	848	869	890	911	931	952	973	994	21	9 2 10 2
209	32	015	035	056	077	098	118	139	160	181	201	21	14 3 14 3
210		222	243	263	284	305	325	346	366	387	408	20	18 4 19 4
211		428	449	469	490	510	531	552	572	593	613	21	23 5 24 5
212		634	654	675	695	715	736	756	777	797	818	20	27 6 29 6
213		838	858	879	899	919	940	960	980	1001	1021	20	32 7 33 7
214	33	041	062	082	102	122	143	163	183	203	224	20	36 8 38 8
215		244	264	284	304	325	345	365	385	405	425	20	41 9 43 9
216		445	465	486	506	526	546	566	586	606	626	20	45 10 46 10
217		646	666	686	706	726	746	766	786	806	826	20	50 11 52 11
218		846	866	885	905	925	945	965	985	1005	1025	20	55 12 57 12
219	34	044	064	084	104	124	143	163	183	203	223	19	59 13 62 13
220		242	262	282	301	321	341	361	380	400	420	19	64 14 67 14
221		439	459	479	498	518	537	557	577	596	616	19	68 15 71 15
222		635	655	674	694	713	733	753	772	792	811	19	73 16 76 16
223		830	850	869	889	908	928	947	967	986	1005	20	77 17 81 17
224	35	025	044	064	083	102	122	141	160	180	199	19	82 18 86 18
225		218	238	257	276	295	315	334	353	372	392	19	86 19 90 19
226		411	430	449	468	488	507	526	545	564	583	20	91 20 95 20
227		603	622	641	660	679	698	717	736	755	774	19	95 21 10 21
228		793	813	832	851	870	889	908	927	946	965	19	10 22
229		984	1003	1021	1040	1059	1078	1097	1116	1135	1154	19	
230	36	173	192	211	229	248	267	286	305	324	342	19	20 20 19
231		361	380	399	418	436	455	474	493	511	530	19	Δ n Δ l Δ n Δ l
232		549	568	586	605	624	642	661	680	698	717	19	5 1 5 1
233		736	754	773	791	810	829	847	866	884	903	19	10 2 11 2
234		922	940	959	977	996	1014	1033	1051	1070	1088	19	15 3 16 3
235	37	107	125	144	162	181	199	218	236	254	273	18	20 4 21 4
236		291	310	328	346	365	383	401	420	438	457	18	25 5 26 5
237		475	493	511	530	548	566	585	603	621	639	18	30 6 32 6
238		658	676	694	712	731	749	767	785	803	822	18	35 7 37 7
239		840	858	876	894	912	931	949	967	985	1003	18	40 8 42 8
240	38	021	039	057	075	093	112	130	148	166	184	18	45 9 47 9
241		202	220	238	256	274	292	310	328	346	364	18	50 10 53 10
242		382	399	417	435	453	471	489	507	525	543	18	55 11 58 11
243		561	578	596	614	632	650	668	686	703	721	18	60 12 63 12
244		739	757	775	792	810	828	846	863	881	899	18	65 13 68 13
245	39	917	934	952	970	987	1005	1023	1041	1058	1076	18	70 14 74 14
246		094	111	129	146	164	182	199	217	235	252	18	75 15 79 15
247		270	287	305	322	340	358	375	393	410	428	17	80 16 84 16
248		445	463	480	498	515	533	550	568	585	602	18	85 17 89 17
249		620	637	655	672	690	707	724	742	759	777	17	90 18 95 18
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

P.P.				N.	Log	0	1	2	3	4	5	6	7	8	9	Δ
18 17				250	39	794	811	829	846	863	881	898	915	933	950	17
Δ n	Δ 1	Δ n	Δ 1	251	40	967	985	1002	1019	1037	1054	1071	1088	1106	1123	17
1 1	2	1 2	2	252		140	157	175	192	209	226	243	261	278	295	17
1 7	3	1 8	3	253		312	329	346	364	381	398	415	432	449	466	17
2 2	4	2 4	4	254		483	500	518	535	552	569	586	603	620	637	17
				255		654	671	688	705	722	739	756	773	790	807	17
2 8	5	2 9	5	256		824	841	858	875	892	909	926	943	960	976	17
3 3	6	3 5	6	257		993	1010	1027	1044	1061	1078	1095	1111	1128	1145	17
3 9	7	4 1	7	258	41	162	179	196	212	229	246	263	280	296	313	17
4 4	8	4 7	8	259		330	347	363	380	397	414	430	447	464	481	16
5 0	9	5 3	9													
5 6	10	5 9	10	260		497	514	531	547	564	581	597	614	631	647	17
6 1	11	6 5	11	261		664	681	697	714	731	747	764	780	797	814	16
6 7	12	7 1	12	262		830	847	863	880	896	913	929	946	963	979	17
7 2	13	7 6	13	263		996	1012	1029	1045	1062	1078	1095	1111	1127	1144	16
7 8	14	8 2	14	264	42	160	177	193	210	226	243	259	275	292	308	17
8 3	15	8 8	15	265		325	341	357	374	390	406	423	439	455	472	16
8 9	16	9 4	16	266		488	504	521	537	553	570	586	602	619	635	16
9 4	17	10	17	267		651	667	684	700	716	732	749	765	781	797	16
10	18			268		813	830	846	862	878	894	911	927	943	959	16
				269		975	991	1008	1024	1040	1056	1072	1088	1104	1120	16
16 15				270	43	136	152	169	185	201	217	233	249	265	281	16
Δ n	Δ 1	Δ n	Δ 1	271		297	313	329	345	361	377	393	409	425	441	16
1 2	3	1 3	3	272		457	473	489	505	521	537	553	569	584	600	16
1 9	4	2 0	4	273		616	632	648	664	680	696	712	727	743	759	16
2 5		2 7		274		775	791	807	823	838	854	870	886	902	917	16
3 1	5	3 3	5													
3 8	6	4 0	6	275		933	949	965	981	996	1012	1028	1044	1059	1075	16
4 4	7	4 7	7	276	44	091	107	122	138	154	170	185	201	217	232	16
5 0	8	5 3	8	277		248	264	279	295	311	326	342	358	373	389	15
5 6	9	6 0	9	278		404	420	436	451	467	483	498	514	529	545	15
6 2	10	6 7	10	279		560	576	592	607	623	638	654	669	685	700	16
6 9	11	7 3	11													
7 5	12	8 0	12	280		716	731	747	762	778	793	809	824	840	855	16
8 1	13	8 7	13	281		871	886	902	917	932	948	963	979	994	1010	15
8 7	14	9 3	14	282	45	025	040	056	071	086	102	117	133	148	163	16
9 4	15	10	15	283		179	194	209	225	240	255	271	286	301	317	15
10	16			284		332	347	362	378	393	408	423	439	454	469	15
				285		484	500	515	530	545	561	576	591	606	621	16
Δ n	Δ 1			286		637	652	667	682	697	712	728	743	758	773	15
7	1			287		788	803	818	834	849	864	879	894	909	924	15
1 4	2			288		939	954	969	984	1000	1015	1030	1045	1060	1075	15
2 1	3			289	46	090	105	120	135	150	165	180	195	210	225	15
2 9	4															
3 6	5			290		240	255	270	285	300	315	330	345	359	374	15
4 3	6			291		389	404	419	434	449	464	479	494	509	523	15
5 0	7			292		538	553	568	583	598	613	627	642	657	672	15
5 7	8			293		687	702	716	731	746	761	776	790	805	820	15
6 4	9			294		835	850	864	879	894	909	923	938	953	967	15
				295		982	997	1012	1026	1041	1056	1070	1085	1100	1114	15
7 1	10			296	47	129	144	159	173	188	202	217	232	246	261	15
7 9	11			297		276	290	305	319	334	349	363	378	392	407	15
8 6	12			298		422	436	451	465	480	494	509	524	538	553	14
9 3	13			299		567	582	596	611	625	640	654	669	683	698	14
10	14															
P.P.				N.	Log	0	1	2	3	4	5	6	7	8	9	Δ

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
300	47	712	727	741	756	770	784	799	813	828	842	15	Δn Δl $\frac{15}{1}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$
301		857	871	885	900	914	929	943	958	972	986	15	
302	48	001	015	029	044	058	073	087	101	116	130	14	
303		144	159	173	187	202	216	230	244	259	273	14	
304		287	302	316	330	344	359	373	387	401	416	14	$\frac{7}{1}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$
305		430	444	458	473	487	501	515	530	544	558	14	$\frac{3}{3}$ $\frac{4}{4}$ $\frac{5}{5}$ $\frac{6}{6}$
306		572	586	601	615	629	643	657	671	686	700	14	$\frac{4}{7}$ $\frac{5}{8}$ $\frac{6}{9}$ $\frac{7}{10}$
307		714	728	742	756	770	785	799	813	827	841	14	$\frac{8}{8}$ $\frac{9}{9}$ $\frac{10}{10}$ $\frac{11}{11}$
308		855	869	883	897	911	926	940	954	968	982	14	$\frac{5}{8}$ $\frac{6}{9}$ $\frac{7}{10}$ $\frac{8}{11}$
309		996	100	124	138	152	166	180	194	208	222	14	$\frac{9}{10}$ $\frac{10}{11}$ $\frac{11}{12}$ $\frac{12}{13}$
310	49	136	150	164	178	192	206	220	234	248	262	14	Δn Δl $\frac{14}{1}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$
311		276	290	304	318	332	346	360	374	388	402	13	
312		415	429	443	457	471	485	499	513	527	541	13	
313		554	568	582	596	610	624	638	651	665	679	14	
314		693	707	721	734	748	762	776	790	803	817	14	$\frac{10}{15}$ $\frac{11}{16}$ $\frac{12}{17}$ $\frac{13}{18}$
315		831	845	859	872	886	900	914	927	941	955	14	Δn Δl $\frac{14}{1}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$
316		969	982	996	100	124	137	151	165	179	192	14	
317	50	106	120	133	147	161	174	188	202	215	229	14	
318		243	256	270	284	297	311	325	338	352	365	14	
319		379	393	406	420	433	447	461	474	488	501	14	$\frac{5}{6}$ $\frac{6}{7}$ $\frac{7}{8}$ $\frac{8}{9}$
320		515	529	542	556	569	583	596	610	623	637	14	Δn Δl $\frac{14}{1}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$
321		651	664	678	691	705	718	732	745	759	772	14	
322		786	799	813	826	840	853	866	880	893	907	13	
323		920	934	947	961	974	987	1001	114	128	141	14	
324	51	055	068	081	095	108	121	135	148	162	175	13	$\frac{9}{10}$ $\frac{10}{11}$ $\frac{11}{12}$ $\frac{12}{13}$
325		188	202	215	228	242	255	268	282	295	308	14	Δn Δl $\frac{14}{1}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$
326		322	335	348	362	375	388	402	415	428	441	14	
327		455	468	481	495	508	521	534	548	561	574	13	
328		587	601	614	627	640	654	667	680	693	706	14	
329		720	733	746	759	772	786	799	812	825	838	13	$\frac{7}{10}$ $\frac{8}{11}$ $\frac{9}{12}$ $\frac{10}{13}$
330		851	865	878	891	904	917	930	943	957	970	13	Δn Δl $\frac{13}{1}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$
331		983	996	1009	1122	1235	1348	1461	1575	1688	1801	13	
332	52	114	127	140	153	166	179	192	205	218	231	13	
333		244	257	270	284	297	310	323	336	349	362	13	
334		375	388	401	414	427	440	453	466	479	492	12	$\frac{10}{14}$ $\frac{11}{15}$ $\frac{12}{16}$ $\frac{13}{17}$
335		504	517	530	543	556	569	582	595	608	621	13	Δn Δl $\frac{13}{1}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$
336		634	647	660	673	686	699	711	724	737	750	13	
337		763	776	789	802	815	827	840	853	866	879	13	
338		892	905	917	930	943	956	969	982	994	1007	13	
339	53	020	033	046	058	071	084	097	110	122	135	13	$\frac{11}{14}$ $\frac{12}{15}$ $\frac{13}{16}$ $\frac{14}{17}$
340		148	161	173	186	199	212	224	237	250	263	12	Δn Δl $\frac{12}{1}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$
341		275	288	301	314	326	339	352	364	377	390	13	
342		403	415	428	441	453	466	479	491	504	517	12	
343		529	542	555	567	580	593	605	618	631	643	13	
344		656	668	681	694	706	719	732	744	757	769	13	$\frac{15}{18}$ $\frac{16}{19}$ $\frac{17}{20}$ $\frac{18}{21}$
345		782	794	807	820	832	845	857	870	882	895	13	Δn Δl $\frac{12}{1}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$
346		908	920	933	945	958	970	983	995	1008	1020	13	
347	54	033	045	058	070	083	095	108	120	133	145	13	
348		158	170	183	195	208	220	233	245	258	270	13	
349		283	295	307	320	332	345	357	370	382	394	13	$\frac{19}{22}$ $\frac{20}{23}$ $\frac{21}{24}$ $\frac{22}{25}$
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
350	54	407	419	432	444	456	469	481	494	506	518	13	
351		531	543	555	568	580	593	605	617	630	642	12	
352		654	667	679	691	704	716	728	741	753	765	12	
353		777	790	802	814	827	839	851	864	876	888	12	
354		900	913	925	937	949	962	974	986	998	111	12	13 Δ n Δ l
355	55	023	035	047	060	072	084	096	108	121	133	12	8 1
356		145	157	169	182	194	206	218	230	242	255	12	16 2
357		267	279	291	303	315	328	340	352	364	376	12	23 3
358		388	400	413	425	437	449	461	473	485	497	12	31 4
359		509	522	534	546	558	570	582	594	606	618	12	38 5
360		630	642	654	666	678	691	703	715	727	739	12	46 6
361		751	763	775	787	799	811	823	835	847	859	12	54 7
362		871	883	895	907	919	931	943	955	967	979	12	62 8
363		991	103	115	127	138	150	162	174	186	198	12	69 9
364	56	110	122	134	146	158	170	182	194	205	217	12	77 10
365		229	241	253	265	277	289	301	312	324	336	12	85 11
366		348	360	372	384	396	407	419	431	443	455	12	92 12
367		467	478	490	502	514	526	538	549	561	573	12	10 13
368		585	597	608	620	632	644	656	667	679	691	12	
369		703	714	726	738	750	761	773	785	797	808	12	
370		820	832	844	855	867	879	891	902	914	926	11	12 Δ n Δ l
371		937	949	961	972	984	996	1008	1019	1031	1043	11	8 1
372	57	054	066	078	089	101	113	124	136	148	159	12	17 2
373		171	183	194	206	217	229	241	252	264	276	11	25 3
374		287	299	310	322	334	345	357	368	380	392	11	33 4
375		403	415	426	438	449	461	473	484	496	507	12	42 5
376		519	530	542	553	565	576	588	600	611	623	11	50 6
377		634	646	657	669	680	692	703	715	726	738	11	58 7
378		749	761	772	784	795	807	818	830	841	852	12	67 8
379		864	875	887	898	910	921	933	944	955	967	11	75 9
380		978	990	101	113	124	135	147	158	170	181	11	83 10
381	58	092	104	115	127	138	149	161	172	184	195	11	92 11
382		206	218	229	240	252	263	274	286	297	309	11	10 12
383		320	331	343	354	365	377	388	399	410	422	11	
384		433	444	456	467	478	490	501	512	524	535	11	
385		546	557	569	580	591	602	614	625	636	647	12	
386		659	670	681	692	704	715	726	737	749	760	11	11 Δ n Δ l
387		771	782	794	805	816	827	838	850	861	872	11	9 1
388		883	894	906	917	928	939	950	961	973	984	11	18 2
389		995	1006	1017	1028	1040	1051	1062	1073	1084	1095	11	27 3
390	59	106	118	129	140	151	162	173	184	195	207	11	36 4
391		218	229	240	251	262	273	284	295	306	318	11	45 5
392		329	340	351	362	373	384	395	406	417	428	11	55 6
393		439	450	461	472	483	494	506	517	528	539	11	64 7
394		550	561	572	583	594	605	616	627	638	649	11	73 8
395		660	671	682	693	704	715	726	737	748	759	11	82 9
396		770	780	791	802	813	824	835	846	857	868	11	91 10
397		879	890	901	912	923	934	945	956	967	977	11	10 11
398		988	999	1010	1021	1032	1043	1054	1065	1076	1086	11	
399	60	097	108	119	130	141	152	163	173	184	195	11	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
400	60	206	217	228	239	249	260	271	282	293	304	10	
401		314	325	336	347	358	369	379	390	401	412	11	
402		423	433	444	455	466	477	487	498	509	520	11	
403		531	541	552	563	574	584	595	606	617	627	11	
404		638	649	660	670	681	692	703	713	724	735	11	
405		746	756	767	778	788	799	810	821	831	842	11	
406		853	863	874	885	895	906	917	927	938	949	10	
407		959	970	981	991	1002	1013	1023	1034	1045	1055	11	
408	61	066	077	087	098	109	119	130	140	151	162	10	
409		172	183	194	204	215	225	236	247	257	268	10	
410		278	289	300	310	321	331	342	352	363	374	10	
411		384	395	405	416	426	437	448	458	469	479	11	
412		490	500	511	521	532	542	553	563	574	584	11	
413		595	606	616	627	637	648	658	669	679	690	10	
414		700	711	721	731	742	752	763	773	784	794	11	
415		805	815	826	836	847	857	868	878	888	899	10	
416		909	920	930	941	951	962	972	982	993	1003	11	
417	62	014	024	034	045	055	066	076	086	097	107	11	
418		118	128	138	149	159	170	180	190	201	211	10	
419		221	232	242	252	263	273	284	294	304	315	10	
420		325	335	346	356	366	377	387	397	408	418	10	
421		428	439	449	459	469	480	490	500	511	521	10	
422		531	542	552	562	572	583	593	603	613	624	10	
423		634	644	655	665	675	685	696	706	716	726	11	
424		737	747	757	767	778	788	798	808	818	829	10	
425		839	849	859	870	880	890	900	910	921	931	10	
426		941	951	961	972	982	992	1002	1012	1022	1033	11	
427	63	043	053	063	073	083	094	104	114	124	134	10	
428		144	155	165	175	185	195	205	215	225	236	10	
429		246	256	266	276	286	296	306	317	327	337	10	
430		347	357	367	377	387	397	407	417	428	438	10	
431		448	458	468	478	488	498	508	518	528	538	10	
432		548	558	568	579	589	599	609	619	629	639	10	
433		649	659	669	679	689	699	709	719	729	739	10	
434		749	759	769	779	789	799	809	819	829	839	10	
435		849	859	869	879	889	899	909	919	929	939	10	
436		949	959	969	979	988	998	1008	1018	1028	1038	10	
437	64	048	058	068	078	088	098	108	118	128	137	10	
438		147	157	167	177	187	197	207	217	227	237	9	
439		246	256	266	276	286	296	306	316	326	335	10	
440		345	355	365	375	385	395	404	414	424	434	10	
441		444	454	464	473	483	493	503	513	523	532	10	
442		542	552	562	572	582	591	601	611	621	631	9	
443		640	650	660	670	680	689	699	709	719	729	9	
444		738	748	758	768	777	787	797	807	816	826	10	
445		836	846	856	865	875	885	895	904	914	924	9	
446		933	943	953	963	972	982	992	1002	1011	1021	10	
447	65	031	040	050	060	070	079	089	099	108	118	10	
448		128	137	147	157	167	176	186	196	205	215	10	
449		225	234	244	254	263	273	283	292	302	312	9	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
450	65	321	331	341	350	360	369	379	389	398	408	10	
451		418	427	437	447	456	466	475	485	495	504	10	
452		514	523	533	543	552	562	571	581	591	600	10	
453		610	619	629	639	648	658	667	677	686	696	10	
454		706	715	725	734	744	753	763	772	782	792	9	
455		801	811	820	830	839	849	858	868	877	887	9	
456		896	906	916	925	935	944	954	963	973	982	10	
457		992	♦01	♦11	♦20	♦30	♦39	♦49	♦58	♦68	♦77	10	
458	66	087	096	106	115	124	134	143	153	162	172	9	
459		181	191	200	210	219	229	238	247	257	266	10	
460		276	285	295	304	314	323	332	342	351	361	9	
461		370	380	389	398	408	417	427	436	445	455	9	
462		464	474	483	492	502	511	521	530	539	549	9	
463		558	567	577	586	596	605	614	624	633	642	10	
464		652	661	671	680	689	699	708	717	727	736	9	
465		745	755	764	773	783	792	801	811	820	829	10	
466		839	848	857	867	876	885	894	904	913	922	10	
467		932	941	950	960	969	978	987	997	♦06	♦15	10	
468	67	025	034	043	052	062	071	080	089	099	108	9	
469		117	127	136	145	154	164	173	182	191	201	9	
470		210	219	228	237	247	256	265	274	284	293	9	
471		302	311	321	330	339	348	357	367	376	385	9	
472		394	403	413	422	431	440	449	459	468	477	9	
473		486	495	504	514	523	532	541	550	560	569	9	
474		578	587	596	605	614	624	633	642	651	660	9	
475		669	679	688	697	706	715	724	733	742	752	9	
476		761	770	779	788	797	806	815	825	834	843	9	
477		852	861	870	879	888	897	906	916	925	934	9	
478		943	952	961	970	979	988	997	♦06	♦15	♦24	10	
479	68	034	043	052	061	070	079	088	097	106	115	9	
480		124	133	142	151	160	169	178	187	196	205	10	
481		215	224	233	242	251	260	269	278	287	296	9	
482		305	314	323	332	341	350	359	368	377	386	9	
483		395	404	413	422	431	440	449	458	467	476	9	
484		485	494	502	511	520	529	538	547	556	565	9	
485		574	583	592	601	610	619	628	637	646	655	9	
486		664	673	681	690	699	708	717	726	735	744	9	
487		753	762	771	780	789	797	806	815	824	833	9	
488		842	851	860	869	878	886	895	904	913	922	9	
489		931	940	949	958	966	975	984	993	♦02	♦11	9	
490	69	020	028	037	046	055	064	073	082	090	099	9	
491		108	117	126	135	144	152	161	170	179	188	9	
492		197	205	214	223	232	241	249	258	267	276	9	
493		285	294	302	311	320	329	338	346	355	364	9	
494		373	381	390	399	408	417	425	434	443	452	9	
495		461	469	478	487	496	504	513	522	531	539	9	
496		548	557	566	574	583	592	601	609	618	627	9	
497		636	644	653	662	671	679	688	697	705	714	9	
498		723	732	740	749	758	767	775	784	793	801	9	
499		810	819	827	836	845	854	862	871	880	888	9	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

10
Δ n Δ l

1 1
2 2
3 3
4 4
5 5
6 6
7 7
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9
Δ n Δ l

1 1
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N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
500	69	897	906	914	923	932	940	949	958	966	975	9	
501		984	992	101	110	118	127	136	144	153	162	8	
502	70	070	079	088	096	105	114	122	131	140	148	9	
503		157	165	174	183	191	200	209	217	226	234	9	
504		243	252	260	269	278	286	295	303	312	321	8	
505		329	338	346	355	364	372	381	389	398	406	9	Δn Δl 1 1 2 2 3 3 4 4
506		415	424	432	441	449	458	467	475	484	492	9	
507		501	509	518	526	535	544	552	561	569	578	8	
508		586	595	603	612	621	629	638	646	655	663	9	
509		672	680	689	697	706	714	723	731	740	749	8	
510		757	766	774	783	791	800	808	817	825	834	8	5 6 6 7 7 8 8 9 9 10
511		842	851	859	868	876	885	893	902	910	919	8	
512		927	935	944	952	961	969	978	986	995	103	9	
513	71	012	020	029	037	046	054	063	071	079	088	8	
514		096	105	113	122	130	139	147	155	164	172	9	
515		181	189	198	206	214	223	231	240	248	257	8	Δn Δl 1 3 2 5 3 8 4 10
516		265	273	282	290	299	307	315	324	332	341	8	
517		349	357	366	374	383	391	399	408	416	425	8	
518		433	441	450	458	466	475	483	492	500	508	9	
519		517	525	533	542	550	559	567	575	584	592	8	
520		600	609	617	625	634	642	650	659	667	675	9	8 Δn Δl 1 3 2 5 3 8 4 10
521		684	692	700	709	717	725	734	742	750	759	8	
522		767	775	784	792	800	809	817	825	834	842	8	
523		850	858	867	875	883	892	900	908	917	925	8	
524		933	941	950	958	966	975	983	991	999	108	8	
525	72	016	024	032	041	049	057	066	074	082	090	9	5 6 6 7 7 8 8 9 9 10
526		099	107	115	123	132	140	148	156	165	173	8	
527		181	189	198	206	214	222	230	239	247	255	8	
528		263	272	280	288	296	304	313	321	329	337	9	
529		346	354	362	370	378	387	395	403	411	419	9	
530		428	436	444	452	460	469	477	485	493	501	8	
531		509	518	526	534	542	550	558	567	575	583	8	
532		591	599	607	616	624	632	640	648	656	665	8	
533		673	681	689	697	705	713	722	730	738	746	8	
534		754	762	770	779	787	795	803	811	819	827	8	
535		835	843	852	860	868	876	884	892	900	908	8	7 Δn Δl 1 4 2 9 3 4 4 7
536		916	925	933	941	949	957	965	973	981	989	8	
537		997	106	114	122	130	138	146	154	162	170	8	
538	73	078	086	094	102	111	119	127	135	143	151	8	
539		159	167	175	183	191	199	207	215	223	231	8	
540		239	247	255	263	272	280	288	296	304	312	8	5 6 6 7 7 8 8 9 9 10
541		320	328	336	344	352	360	368	376	384	392	8	
542		400	408	416	424	432	440	448	456	464	472	8	
543		480	488	496	504	512	520	528	536	544	552	8	
544		560	568	576	584	592	600	608	616	624	632	8	
545		640	648	656	664	672	679	687	695	703	711	8	
546		719	727	735	743	751	759	767	775	783	791	8	
547		799	807	815	823	830	838	846	854	862	870	8	
548		878	886	894	902	910	918	926	933	941	949	8	
549		957	965	973	981	989	997	105	113	120	128	8	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
550	74	036	044	052	060	068	076	084	092	099	107	8	
551		115	123	131	139	147	155	162	170	178	186	8	
552		194	202	210	218	225	233	241	249	257	265	8	
553		273	280	288	296	304	312	320	327	335	343	8	
554		351	359	367	374	382	390	398	406	414	421	8	
555		429	437	445	453	461	468	476	484	492	500	7	
556		507	515	523	531	539	547	554	562	570	578	8	
557		586	593	601	609	617	624	632	640	648	656	7	
558		663	671	679	687	695	702	710	718	726	733	8	
559		741	749	757	764	772	780	788	796	803	811	8	
560		819	827	834	842	850	858	865	873	881	889	7	
561		896	904	912	920	927	935	943	950	958	966	8	
562		974	981	989	997	005	012	020	028	035	043	8	
563	75	051	059	066	074	082	089	097	105	113	120	8	
564		128	136	143	151	159	166	174	182	189	197	8	$\Delta n \Delta 1$ 8 13 1 25 2 38 3 50 4 63 5 75 6 88 7 10 8
565		205	213	220	228	236	243	251	259	266	274	8	
566		282	289	297	305	312	320	328	335	343	351	7	
567		358	366	374	381	389	397	404	412	420	427	8	
568		435	442	450	458	465	473	481	488	496	504	7	
569		511	519	526	534	542	549	557	565	572	580	7	
570		587	595	603	610	618	626	633	641	648	656	8	
571		664	671	679	686	694	702	709	717	724	732	8	
572		740	747	755	762	770	778	785	793	800	808	7	
573		815	823	831	838	846	853	861	868	876	884	7	
574		891	899	906	914	921	929	937	944	952	959	8	
575		967	974	982	989	997	005	012	020	027	035	7	
576	76	042	050	057	065	072	080	087	095	103	110	8	
577		118	125	133	140	148	155	163	170	178	185	8	
578		193	200	208	215	223	230	238	245	253	260	8	
579		268	275	283	290	298	305	313	320	328	335	8	
580		343	350	358	365	373	380	388	395	403	410	8	
581		418	425	433	440	448	455	462	470	477	485	7	
582		492	500	507	515	522	530	537	545	552	559	7	
583		567	574	582	589	597	604	612	619	626	634	8	
584		641	649	656	664	671	678	686	693	701	708	8	$\Delta n \Delta 1$ 7 14 1 29 2 43 3 57 4 71 5 86 6 10 7
585		716	723	730	738	745	753	760	768	775	782	8	
586		790	797	805	812	819	827	834	842	849	856	8	
587		864	871	879	886	893	901	908	916	923	930	8	
588		938	945	953	960	967	975	982	989	997	004	8	
589	77	012	019	026	034	041	048	056	063	070	078	7	
590		085	093	100	107	115	122	129	137	144	151	8	
591		159	166	173	181	188	195	203	210	217	225	7	
592		232	240	247	254	262	269	276	283	291	298	7	
593		305	313	320	327	335	342	349	357	364	371	8	
594		379	386	393	401	408	415	422	430	437	444	8	
595		452	459	466	474	481	488	495	503	510	517	8	
596		525	532	539	546	554	561	568	576	583	590	7	
597		597	605	612	619	627	634	641	648	656	663	7	
598		670	677	685	692	699	706	714	721	728	735	8	
599		743	750	757	764	772	779	786	793	801	808	7	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
600	77	815	822	830	837	844	851	859	866	873	880	7	Δn $\Delta 1$ 1.3 1 2.5 2 3.8 3 5.0 4
601		887	895	902	909	916	924	931	938	945	952	8	
602		960	967	974	981	988	996	1003	1010	1017	1025	7	
603	78	032	039	046	053	061	068	075	082	089	097	7	
604		104	111	118	125	132	140	147	154	161	168	8	Δn $\Delta 1$ 1.3 1 2.5 2 3.8 3 5.0 4
605		176	183	190	197	204	211	219	226	233	240	7	
606		247	254	262	269	276	283	290	297	305	312	7	
607		319	326	333	340	347	355	362	369	376	383	7	
608		390	398	405	412	419	426	433	440	447	455	7	Δn $\Delta 1$ 1.3 1 2.5 2 3.8 3 5.0 4
609		462	469	476	483	490	497	504	512	519	526	7	
610		533	540	547	554	561	569	576	583	590	597	7	
611		604	611	618	625	633	640	647	654	661	668	7	
612		675	682	689	696	704	711	718	725	732	739	7	Δn $\Delta 1$ 1.3 1 2.5 2 3.8 3 5.0 4
613		746	753	760	767	774	781	789	796	803	810	7	
614		817	824	831	838	845	852	859	866	873	880	8	
615		888	895	902	909	916	923	930	937	944	951	7	
616		958	965	972	979	986	993	1000	1007	1014	1021	8	Δn $\Delta 1$ 1.3 1 2.5 2 3.8 3 5.0 4
617	79	029	036	043	050	057	064	071	078	085	092	7	
618		099	106	113	120	127	134	141	148	155	162	7	
619		169	176	183	190	197	204	211	218	225	232	7	
620		239	246	253	260	267	274	281	288	295	302	7	Δn $\Delta 1$ 1.3 1 2.5 2 3.8 3 5.0 4
621		309	316	323	330	337	344	351	358	365	372	7	
622		379	386	393	400	407	414	421	428	435	442	7	
623		449	456	463	470	477	484	491	498	505	511	7	
624		518	525	532	539	546	553	560	567	574	581	7	Δn $\Delta 1$ 1.3 1 2.5 2 3.8 3 5.0 4
625		588	595	602	609	616	623	630	637	644	650	7	
626		657	664	671	678	685	692	699	706	713	720	7	
627		727	734	741	748	754	761	768	775	782	789	7	
628		796	803	810	817	824	831	837	844	851	858	7	Δn $\Delta 1$ 1.3 1 2.5 2 3.8 3 5.0 4
629		865	872	879	886	893	900	906	913	920	927	7	
630		934	941	948	955	962	969	975	982	989	996	7	
631	80	003	010	017	024	030	037	044	051	058	065	7	
632		072	079	085	092	099	106	113	120	127	134	6	Δn $\Delta 1$ 1.3 1 2.5 2 3.8 3 5.0 4
633		140	147	154	161	168	175	182	188	195	202	7	
634		209	216	223	229	236	243	250	257	264	271	6	
635		277	284	291	298	305	312	318	325	332	339	7	
636		346	353	359	366	373	380	387	393	400	407	7	Δn $\Delta 1$ 1.3 1 2.5 2 3.8 3 5.0 4
637		414	421	428	434	441	448	455	462	468	475	7	
638		482	489	496	502	509	516	523	530	536	543	7	
639		550	557	564	570	577	584	591	598	604	611	7	
640		618	625	632	638	645	652	659	665	672	679	7	Δn $\Delta 1$ 1.3 1 2.5 2 3.8 3 5.0 4
641		686	693	699	706	713	720	726	733	740	747	7	
642		754	760	767	774	781	787	794	801	808	814	7	
643		821	828	835	841	848	855	862	868	875	882	7	
644		889	895	902	909	916	922	929	936	943	949	7	Δn $\Delta 1$ 1.3 1 2.5 2 3.8 3 5.0 4
645	81	956	963	969	976	983	990	996	1003	1010	1017	6	
646		023	030	037	043	050	057	064	070	077	084	6	
647		090	097	104	111	117	124	131	137	144	151	7	
648		158	164	171	178	184	191	198	204	211	218	6	Δn $\Delta 1$ 1.3 1 2.5 2 3.8 3 5.0 4
649		224	231	238	245	251	258	265	271	278	285	6	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
650	81	291	298	305	311	318	325	331	338	345	351	7	
651		358	365	371	378	385	391	398	405	411	418	7	
652		425	431	438	445	451	458	465	471	478	485	6	
653		491	498	505	511	518	525	531	538	544	551	7	
654		558	564	571	578	584	591	598	604	611	617	7	
655		624	631	637	644	651	657	664	671	677	684	6	
656		690	697	704	710	717	723	730	737	743	750	7	
657		757	763	770	776	783	790	796	803	809	816	7	
658		823	829	836	842	849	856	862	869	875	882	7	
659		889	895	902	908	915	921	928	935	941	948	6	
660		954	961	968	974	981	987	994	1000	1007	1014	6	
661	82	020	027	033	040	046	053	060	066	073	079	7	Δn 7
662		086	092	099	105	112	119	125	132	138	145	6	Δl 1
663		151	158	164	171	178	184	191	197	204	210	7	1.4 1
664		217	223	230	236	243	249	256	263	269	276	6	2.9 2
665		282	289	295	302	308	315	321	328	334	341	6	4.3 3
666		347	354	360	367	373	380	387	393	400	406	7	6.7 4
667		413	419	426	432	439	445	452	458	465	471	7	7.1 5
668		478	484	491	497	504	510	517	523	530	536	7	8.6 6
669		543	549	556	562	569	575	582	588	595	601	6	10 7
670		607	614	620	627	633	640	646	653	659	666	6	
671		672	679	685	692	698	705	711	718	724	730	7	
672		737	743	750	756	763	769	776	782	789	795	7	
673		802	808	814	821	827	834	840	847	853	860	6	
674		866	872	879	885	892	898	905	911	918	924	6	
675		930	937	943	950	956	963	969	975	982	988	7	
676		995	1001	1008	1014	1020	1027	1033	1040	1046	1052	7	
677	83	059	065	072	078	085	091	097	104	110	117	6	
678		123	129	136	142	149	155	161	168	174	181	6	
679		187	193	200	206	213	219	225	232	238	245	6	
680		251	257	264	270	276	283	289	296	302	308	7	
681		315	321	327	334	340	347	353	359	366	372	6	
682		378	385	391	398	404	410	417	423	429	436	6	
683		442	448	455	461	467	474	480	487	493	499	7	
684		506	512	518	525	531	537	544	550	556	563	6	
685		569	575	582	588	594	601	607	613	620	626	6	
686		632	639	645	651	658	664	670	677	683	689	7	
687		696	702	708	715	721	727	734	740	746	753	6	
688		759	765	771	778	784	790	797	803	809	816	6	
689		822	828	835	841	847	853	860	866	872	879	6	
690		885	891	897	904	910	916	923	929	935	942	6	
691		948	954	960	967	973	979	985	992	998	1004	7	
692	84	011	017	023	029	036	042	048	055	061	067	6	
693		073	080	086	092	098	105	111	117	123	130	6	
694		136	142	148	155	161	167	173	180	186	192	6	
695		198	205	211	217	223	230	236	242	248	255	6	
696		261	267	273	280	286	292	298	305	311	317	6	
697		323	330	336	342	348	354	361	367	373	379	7	
698		386	392	398	404	410	417	423	429	435	442	6	
699		448	454	460	466	473	479	485	491	497	504	6	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

FIVE-PLACE LOGARITHMS

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
700	84	510	516	522	528	535	541	547	553	559	566	6	
701		572	578	584	590	597	603	609	615	621	628	6	
702		634	640	646	652	658	665	671	677	683	689	7	
703		696	702	708	714	720	726	733	739	745	751	6	
704		757	763	770	776	782	788	794	800	807	813	6	
705		819	825	831	837	844	850	856	862	868	874	6	
706		880	887	893	899	905	911	917	924	930	936	6	
707		942	948	954	960	967	973	979	985	991	997	6	
708	85	003	009	016	022	028	034	040	046	052	058	7	Δn Δl
709		065	071	077	083	089	095	101	107	114	120	6	Δn Δl
710		126	132	138	144	150	156	163	169	175	181	6	Δn Δl
711		187	193	199	205	211	217	224	230	236	242	6	Δn Δl
712		248	254	260	266	272	278	285	291	297	303	6	Δn Δl
713		309	315	321	327	333	339	345	352	358	364	6	Δn Δl
714		370	376	382	388	394	400	406	412	418	425	6	Δn Δl
715		431	437	443	449	455	461	467	473	479	485	6	
716		491	497	503	509	516	522	528	534	540	546	6	
717		552	558	564	570	576	582	588	594	600	606	6	
718		612	618	625	631	637	643	649	655	661	667	6	
719		673	679	685	691	697	703	709	715	721	727	6	
720		733	739	745	751	757	763	769	775	781	788	6	
721		794	800	806	812	818	824	830	836	842	848	6	
722		854	860	866	872	878	884	890	896	902	908	6	
723		914	920	926	932	938	944	950	956	962	968	6	
724		974	980	986	992	998	004	010	016	022	028	6	
725	86	034	040	046	052	058	064	070	076	082	088	6	Δn Δl
726		094	100	106	112	118	124	130	136	141	147	6	Δn Δl
727		153	159	165	171	177	183	189	195	201	207	6	Δn Δl
728		213	219	225	231	237	243	249	255	261	267	6	Δn Δl
729		273	279	285	291	297	303	308	314	320	326	6	Δn Δl
730		332	338	344	350	356	362	368	374	380	386	6	
731		392	398	404	410	415	421	427	433	439	445	6	
732		451	457	463	469	475	481	487	493	499	504	6	
733		510	516	522	528	534	540	546	552	558	564	6	
734		570	576	581	587	593	599	605	611	617	623	6	
735		629	635	641	646	652	658	664	670	676	682	6	
736		688	694	700	705	711	717	723	729	735	741	6	
737		747	753	759	764	770	776	782	788	794	800	6	
738		806	812	817	823	829	835	841	847	853	859	5	
739		864	870	876	882	888	894	900	906	911	917	6	
740		923	929	935	941	947	953	958	964	970	976	6	
741	87	982	988	994	999	005	011	017	023	029	035	5	Δn Δl
742		040	046	052	058	064	070	075	081	087	093	6	Δn Δl
743		099	105	111	116	122	128	134	140	146	151	6	Δn Δl
744		157	163	169	175	181	186	192	198	204	210	6	Δn Δl
745		216	221	227	233	239	245	251	256	262	268	6	
746		274	280	286	291	297	303	309	315	320	326	6	
747		332	338	344	349	355	361	367	373	379	384	6	
748		390	396	402	408	413	419	425	431	437	442	6	
749		448	454	460	466	471	477	483	489	495	500	6	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
750	87	506	512	518	523	529	535	541	547	552	558	6	
751		564	570	576	581	587	593	599	604	610	616	6	
752		622	628	633	639	645	651	656	662	668	674	5	
753		679	685	691	697	703	708	714	720	726	731	6	
754		737	743	749	754	760	766	772	777	783	789	6	
755		795	800	806	812	818	823	829	835	841	846	6	
756		852	858	864	869	875	881	887	892	898	904	6	
757		910	915	921	927	933	938	944	950	955	961	6	
758		967	973	978	984	990	996	♦01	♦07	♦13	♦18	6	
759	88	024	030	036	041	047	053	058	064	070	076	5	
760		081	087	093	098	104	110	116	121	127	133	5	
761		138	144	150	156	161	167	173	178	184	190	5	
762		195	201	207	213	218	224	230	235	241	247	5	
763		252	258	264	270	275	281	287	292	298	304	5	
764		309	315	321	326	332	338	343	349	355	360	6	
765		366	372	377	383	389	395	400	406	412	417	6	
766		423	429	434	440	446	451	457	463	468	474	6	
767		480	485	491	497	502	508	513	519	525	530	6	
768		536	542	547	553	559	564	570	576	581	587	6	
769		593	598	604	610	615	621	627	632	638	643	6	
770		649	655	660	666	672	677	683	689	694	700	5	
771		705	711	717	722	728	734	739	745	750	756	5	
772		762	767	773	779	784	790	795	801	807	812	6	
773		818	824	829	835	840	846	852	857	863	868	6	
774		874	880	885	891	897	902	908	913	919	925	5	
775		930	936	941	947	953	958	964	969	975	981	5	
776		986	992	997	♦03	♦09	♦14	♦20	♦25	♦31	♦37	5	
777	89	042	048	053	059	064	070	076	081	087	092	6	
778		098	104	109	115	120	126	131	137	143	148	6	
779		154	159	165	170	176	182	187	193	198	204	5	
780		209	215	221	226	232	237	243	248	254	260	5	
781		265	271	276	282	287	293	298	304	310	315	6	
782		321	326	332	337	343	348	354	360	365	371	5	
783		376	382	387	393	398	404	409	415	421	426	6	
784		432	437	443	448	454	459	465	470	476	481	6	
785		487	492	498	504	509	515	520	526	531	537	5	
786		542	548	553	559	564	570	575	581	586	592	5	
787		597	603	609	614	620	625	631	636	642	647	6	
788		653	658	664	669	675	680	686	691	697	702	6	
789		708	713	719	724	730	735	741	746	752	757	6	
790		763	768	774	779	785	790	796	801	807	812	6	
791		818	823	829	834	840	845	851	856	862	867	6	
792		873	878	883	889	894	900	905	911	916	922	5	
793		927	933	938	944	949	955	960	966	971	977	5	
794		982	988	993	998	♦04	♦09	♦15	♦20	♦26	♦31	6	
795	90	037	042	048	053	059	064	069	075	080	086	5	
796		091	097	102	108	113	119	124	129	135	140	6	
797		146	151	157	162	168	173	179	184	189	195	5	
798		200	206	211	217	222	227	233	238	244	249	6	
799		255	260	266	271	276	282	287	293	298	304	5	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

Δ n	Δ 1
17	1
33	2
50	3
67	4
83	5
10	6

Δ n	Δ 1
20	1
40	2
60	3
80	4
10	5

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
800	90	309	314	320	325	331	336	342	347	352	358	5	Δn Δl $\frac{6}{1}$ $\frac{17}{2}$ $\frac{33}{3}$ $\frac{50}{4}$ $\frac{67}{5}$ $\frac{83}{6}$
801		363	369	374	380	385	390	396	401	407	412	5	
802		417	423	428	434	439	445	450	455	461	466	6	
803		472	477	482	488	493	499	504	509	515	520	6	
804		526	531	536	542	547	553	558	563	569	574	6	
805		580	585	590	596	601	607	612	617	623	628	6	
806		634	639	644	650	655	660	666	671	677	682	5	
807		687	693	698	703	709	714	720	725	730	736	5	
808		741	747	752	757	763	768	773	779	784	789	6	
809		795	800	806	811	816	822	827	832	838	843	6	
810		849	854	859	865	870	875	881	886	891	897	5	
811		902	907	913	918	924	929	934	940	945	950	6	
812		956	961	966	972	977	982	988	993	998	1004	5	
813	91	009	014	020	025	030	036	041	046	052	057	5	
814		062	068	073	078	084	089	094	100	105	110	6	
815		116	121	126	132	137	142	148	153	158	164	5	
816		169	174	180	185	190	196	201	206	212	217	5	
817		222	228	233	238	243	249	254	259	265	270	5	
818		275	281	286	291	297	302	307	312	318	323	5	
819		328	334	339	344	350	355	360	365	371	376	5	
820		381	387	392	397	403	408	413	418	424	429	5	Δn Δl $\frac{5}{1}$ $\frac{20}{2}$ $\frac{40}{3}$ $\frac{60}{4}$ $\frac{80}{5}$
821		434	440	445	450	455	461	466	471	477	482	5	
822		487	492	498	503	508	514	519	524	529	535	5	
823		540	545	551	556	561	566	572	577	582	587	6	
824		593	598	603	609	614	619	624	630	635	640	5	
825		645	651	656	661	666	672	677	682	687	693	5	
826		698	703	709	714	719	724	730	735	740	745	6	
827		751	756	761	766	772	777	782	787	793	798	5	
828		803	808	814	819	824	829	834	840	845	850	5	
829		855	861	866	871	876	882	887	892	897	903	5	
830		908	913	918	924	929	934	939	944	950	955	5	
831		960	965	971	976	981	986	991	997	1002	1007	5	
832	92	012	018	023	028	033	038	044	049	054	059	6	
833		065	070	075	080	085	091	096	101	106	111	6	
834		117	122	127	132	137	143	148	153	158	163	6	
835		169	174	179	184	189	195	200	205	210	215	6	
836		221	226	231	236	241	247	252	257	262	267	6	
837		273	278	283	288	293	298	304	309	314	319	5	
838		324	330	335	340	345	350	355	361	366	371	5	
839		376	381	387	392	397	402	407	412	418	423	5	
840		428	433	438	443	449	454	459	464	469	474	6	Δn Δl $\frac{5}{1}$ $\frac{20}{2}$ $\frac{40}{3}$ $\frac{60}{4}$ $\frac{80}{5}$
841		480	485	490	495	500	505	511	516	521	526	5	
842		531	536	542	547	552	557	562	567	572	578	5	
843		583	588	593	598	603	609	614	619	624	629	5	
844		634	639	645	650	655	660	665	670	675	681	5	
845		686	691	696	701	706	711	716	722	727	732	5	
846		737	742	747	752	758	763	768	773	778	783	5	
847		788	793	799	804	809	814	819	824	829	834	6	
848		840	845	850	855	860	865	870	875	881	886	5	
849		891	896	901	906	911	916	921	927	932	937	5	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
850	92	942	947	952	957	962	967	973	978	983	988	5	
851		993	998	1003	1008	1013	1018	1024	1029	1034	1039	5	
852	93	044	049	054	059	064	069	075	080	085	090	5	
853		095	100	105	110	115	120	125	131	136	141	5	
854		146	151	156	161	166	171	176	181	186	192	5	
855		197	202	207	212	217	222	227	232	237	242	5	Δn Δl
856		247	252	258	263	268	273	278	283	288	293	5	
857		298	303	308	313	318	323	328	334	339	344	5	
858		349	354	359	364	369	374	379	384	389	394	5	
859		399	404	409	414	420	425	430	435	440	445	5	
860		450	455	460	465	470	475	480	485	490	495	5	Δn Δl
861		500	505	510	515	520	526	531	536	541	546	5	
862		551	556	561	566	571	576	581	586	591	596	5	
863		601	606	611	616	621	626	631	636	641	646	5	
864		651	656	661	666	671	676	682	687	692	697	5	
865		702	707	712	717	722	727	732	737	742	747	5	Δn Δl
866		752	757	762	767	772	777	782	787	792	797	5	
867		802	807	812	817	822	827	832	837	842	847	5	
868		852	857	862	867	872	877	882	887	892	897	5	
869		902	907	912	917	922	927	932	937	942	947	5	
870		952	957	962	967	972	977	982	987	992	997	5	Δn Δl
871	94	002	007	012	017	022	027	032	037	042	047	5	
872		052	057	062	067	072	077	082	086	091	096	5	
873		101	106	111	116	121	126	131	136	141	146	5	
874		151	156	161	166	171	176	181	186	191	196	5	
875		201	206	211	216	221	226	231	236	240	245	5	Δn Δl
876		250	255	260	265	270	275	280	285	290	295	5	
877		300	305	310	315	320	325	330	335	340	345	4	
878		349	354	359	364	369	374	379	384	389	394	5	
879		399	404	409	414	419	424	429	433	438	443	5	
880		448	453	458	463	468	473	478	483	488	493	5	Δn Δl
881		498	503	507	512	517	522	527	532	537	542	5	
882		547	552	557	562	567	571	576	581	586	591	5	
883		596	601	606	611	616	621	626	630	635	640	5	
884		645	650	655	660	665	670	675	680	685	689	5	
885		694	699	704	709	714	719	724	729	734	738	5	Δn Δl
886		743	748	753	758	763	768	773	778	783	787	5	
887		792	797	802	807	812	817	822	827	832	836	5	
888		841	846	851	856	861	866	871	876	880	885	5	
889		890	895	900	905	910	915	919	924	929	934	5	
890		939	944	949	954	959	963	968	973	978	983	5	Δn Δl
891	95	988	993	998	1002	1007	1012	1017	1022	1027	1032	4	
892		036	041	046	051	056	061	066	071	075	080	5	
893		085	090	095	100	105	109	114	119	124	129	5	
894		134	139	143	148	153	158	163	168	173	177	5	
895		182	187	192	197	202	207	211	216	221	226	5	Δn Δl
896		231	236	240	245	250	255	260	265	270	274	5	
897		279	284	289	294	299	303	308	313	318	323	5	
898		328	332	337	342	347	352	357	361	366	371	5	
899		376	381	386	390	395	400	405	410	415	419	5	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
900	95	424	429	434	439	444	448	453	458	463	468	4	
901		472	477	482	487	492	497	501	506	511	516	5	
902		521	525	530	535	540	545	550	554	559	564	5	
903		569	574	578	583	588	593	598	602	607	612	5	
904		617	622	626	631	636	641	646	650	655	660	5	
905		665	670	674	679	684	689	694	698	703	708	5	
906		713	718	722	727	732	737	742	746	751	756	5	
907		761	766	770	775	780	785	789	794	799	804	5	
908		809	813	818	823	828	832	837	842	847	852	4	
909		856	861	866	871	875	880	885	890	895	899	5	
910		904	909	914	918	923	928	933	938	942	947	5	
911		952	957	961	966	971	976	980	985	990	995	4	
912		999	004	009	014	019	023	028	033	038	042	5	
913	96	047	052	057	061	066	071	076	080	085	090	5	
914		095	099	104	109	114	118	123	128	133	137	5	Δn 5
915		142	147	152	156	161	166	171	175	180	185	5	20 1
916		190	194	199	204	209	213	218	223	227	232	5	40 2
917		237	242	246	251	256	261	265	270	275	280	4	60 3
918		284	289	294	298	303	308	313	317	322	327	5	80 4
919		332	336	341	346	350	355	360	365	369	374	5	10 5
920		379	384	388	393	398	402	407	412	417	421	5	
921		426	431	435	440	445	450	454	459	464	468	5	
922		473	478	483	487	492	497	501	506	511	515	5	
923		520	525	530	534	539	544	548	553	558	562	5	
924		567	572	577	581	586	591	595	600	605	609	5	
925		614	619	624	628	633	638	642	647	652	656	5	
926		661	666	670	675	680	685	689	694	699	703	5	
927		708	713	717	722	727	731	736	741	745	750	5	
928		755	759	764	769	774	778	783	788	792	797	5	
929		802	806	811	816	820	825	830	834	839	844	4	
930		848	853	858	862	867	872	876	881	886	890	5	
931		895	900	904	909	914	918	923	928	932	937	5	
932		942	946	951	956	960	965	970	974	979	984	4	
933		988	993	997	002	007	011	016	021	025	030	5	
934	97	035	039	044	049	053	058	063	067	072	077	4	Δn 4
935		081	086	090	095	100	104	109	114	118	123	5	25 1
936		128	132	137	142	146	151	155	160	165	169	5	50 2
937		174	179	183	188	192	197	202	206	211	216	4	75 3
938		220	225	230	234	239	243	248	253	257	262	5	10 4
939		267	271	276	280	285	290	294	299	304	308	5	
940		313	317	322	327	331	336	340	345	350	354	5	
941		359	364	368	373	377	382	387	391	396	400	5	
942		405	410	414	419	424	428	433	437	442	447	4	
943		451	456	460	465	470	474	479	483	488	493	4	
944		497	502	506	511	516	520	525	529	534	539	4	
945		543	548	552	557	562	566	571	575	580	585	4	
946		589	594	598	603	607	612	617	621	626	630	5	
947		635	640	644	649	653	658	663	667	672	676	5	
948		681	685	690	695	699	704	708	713	717	722	5	
949		727	731	736	740	745	749	754	759	763	768	4	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
950	97	772	777	782	786	791	795	800	804	809	813	5	
951		818	823	827	832	836	841	845	850	855	859	5	
952		864	868	873	877	882	886	891	896	900	905	4	
953		909	914	918	923	928	932	937	941	946	950	5	
954		955	959	964	968	973	978	982	987	991	996	4	
955	98	000	005	009	014	019	023	028	032	037	041	5	
956		046	050	055	059	064	068	073	078	082	087	4	
957		091	096	100	105	109	114	118	123	127	132	5	
958		137	141	146	150	155	159	164	168	173	177	5	
959		182	186	191	195	200	204	209	214	218	223	4	
960		227	232	236	241	245	250	254	259	263	268	4	
961		272	277	281	286	290	295	299	304	308	313	5	
962		318	322	327	331	336	340	345	349	354	358	5	
963		363	367	372	376	381	385	390	394	399	403	5	
964		408	412	417	421	426	430	435	439	444	448	5	
965		453	457	462	466	471	475	480	484	489	493	5	
966		498	502	507	511	516	520	525	529	534	538	5	
967		543	547	552	556	561	565	570	574	579	583	5	
968		588	592	597	601	605	610	614	619	623	628	4	
969		632	637	641	646	650	655	659	664	668	673	4	
970		677	682	686	691	695	700	704	709	713	717	5	
971		722	726	731	735	740	744	749	753	758	762	5	
972		767	771	776	780	784	789	793	798	802	807	4	
973		811	816	820	825	829	834	838	843	847	851	5	
974		856	860	865	869	874	878	883	887	892	896	4	
975		900	905	909	914	918	923	927	932	936	941	4	
976		945	949	954	958	963	967	972	976	981	985	4	
977		989	994	998	003	007	012	016	021	025	029	5	
978	99	034	038	043	047	052	056	061	065	069	074	4	
979		078	083	087	092	096	100	105	109	114	118	5	
980		123	127	131	136	140	145	149	154	158	162	5	
981		167	171	176	180	185	189	193	198	202	207	4	
982		211	216	220	224	229	233	238	242	247	251	4	
983		255	260	264	269	273	277	282	286	291	295	5	
984		300	304	308	313	317	322	326	330	335	339	5	
985		344	348	352	357	361	366	370	374	379	383	5	
986		388	392	396	401	405	410	414	419	423	427	5	
987		432	436	441	445	449	454	458	463	467	471	5	
988		476	480	484	489	493	498	502	506	511	515	5	
989		520	524	528	533	537	542	546	550	555	559	5	
990		564	568	572	577	581	585	590	594	599	603	4	
991		607	612	616	621	625	629	634	638	642	647	4	
992		651	656	660	664	669	673	677	682	686	691	4	
993		695	699	704	708	712	717	721	726	730	734	5	
994		739	743	747	752	756	760	765	769	774	778	4	
995		782	787	791	795	800	804	808	813	817	822	4	
996		826	830	835	839	843	848	852	856	861	865	5	
997		870	874	878	883	887	891	896	900	904	909	4	
998		913	917	922	926	930	935	939	944	948	952	5	
999		957	961	965	970	974	978	983	987	991	996	4	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

Δ n	Δ l
2.0	1
4.0	2
6.0	3
8.0	4
10	5

Δ n	Δ l
2.5	1
5.0	2
7.5	3
10	4

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
1000	∞	000	004	009	013	017	022	026	030	035	039	4	
1001		043	048	052	056	061	065	069	074	078	082	5	
1002		087	091	095	100	104	108	113	117	121	126	4	
1003		130	134	139	143	147	152	156	160	165	169	4	
1004		173	178	182	186	191	195	199	204	208	212	5	
1005		217	221	225	230	234	238	243	247	251	255	5	
1006		260	264	268	273	277	281	286	290	294	299	4	
1007		303	307	312	316	320	325	329	333	337	342	4	
1008		346	350	355	359	363	368	372	376	381	385	4	
1009		389	393	398	402	406	411	415	419	424	428	4	
1010		432	436	441	445	449	454	458	462	467	471	4	
1011		475	479	484	488	492	497	501	505	509	514	4	
1012		518	522	527	531	535	540	544	548	552	557	4	
1013		561	565	570	574	578	582	587	591	595	600	4	
1014		604	608	612	617	621	625	629	634	638	642	5	Δn Δl
1015		647	651	655	659	664	668	672	677	681	685	4	2.0 1
1016		689	694	698	702	706	711	715	719	724	728	4	4.0 2
1017		732	736	741	745	749	753	758	762	766	771	4	6.0 3
1018		775	779	783	788	792	796	800	805	809	813	4	8.0 4
1019		817	822	826	830	834	839	843	847	852	856	4	10 5
1020		860	864	869	873	877	881	886	890	894	898	5	
1021		903	907	911	915	920	924	928	932	937	941	4	
1022		945	949	954	958	962	966	971	975	979	983	5	
1023		988	992	996	∞00	∞05	∞09	∞13	∞17	∞22	∞26	4	
1024	01	030	034	038	043	047	051	055	060	064	068	4	
1025		072	077	081	085	089	094	098	102	106	111	4	
1026		115	119	123	127	132	136	140	144	149	153	4	
1027		157	161	166	170	174	178	182	187	191	195	4	
1028		199	204	208	212	216	220	225	229	233	237	5	
1029		242	246	250	254	258	263	267	271	275	280	4	
1030		284	288	292	296	301	305	309	313	317	322	4	
1031		326	330	334	339	343	347	351	355	360	364	4	
1032		368	372	376	381	385	389	393	397	402	406	4	
1033		410	414	418	423	427	431	435	439	444	448	4	Δn Δl
1034		452	456	460	465	469	473	477	481	486	490	4	2.5 1
1035		494	498	502	507	511	515	519	523	528	532	4	5.0 2
1036		536	540	544	549	553	557	561	565	569	574	4	7.5 3
1037		578	582	586	590	595	599	603	607	611	616	4	10 4
1038		620	624	628	632	636	641	645	649	653	657	5	
1039		662	666	670	674	678	682	687	691	695	699	4	
1040		703	708	712	716	720	724	728	733	737	741	4	
1041		745	749	753	758	762	766	770	774	778	783	4	
1042		787	791	795	799	803	808	812	816	820	824	4	
1043		828	833	837	841	845	849	853	858	862	866	4	
1044		870	874	878	883	887	891	895	899	903	907	5	
1045		912	916	920	924	928	932	937	941	945	949	4	
1046		953	957	961	966	970	974	978	982	986	991	4	
1047		995	999	∞03	∞07	∞11	∞15	∞20	∞24	∞28	∞32	4	
1048	02	036	040	044	049	053	057	061	065	069	073	5	
1049		078	082	086	090	094	098	102	107	111	115	4	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
1050	02	119	123	127	131	135	140	144	148	152	156	4	
1051		160	164	169	173	177	181	185	189	193	197	5	
1052		202	206	210	214	218	222	226	230	235	239	4	
1053		243	247	251	255	259	263	268	272	276	280	4	
1054		284	288	292	296	301	305	309	313	317	321	4	
1055		325	329	333	338	342	346	350	354	358	362	4	
1056		366	371	375	379	383	387	391	395	399	403	4	
1057		407	412	416	420	424	428	432	436	440	444	5	
1058		449	453	457	461	465	469	473	477	481	485	5	
1059		490	494	498	502	506	510	514	518	522	526	5	
1060		531	535	539	543	547	551	555	559	563	567	5	
1061		572	576	580	584	588	592	596	600	604	608	4	
1062		612	617	621	625	629	633	637	641	645	649	4	
1063		653	657	661	666	670	674	678	682	686	690	4	
1064		694	698	702	706	710	715	719	723	727	731	4	
1065		735	739	743	747	751	755	759	763	768	772	4	
1066		776	780	784	788	792	796	800	804	808	812	4	
1067		816	821	825	829	833	837	841	845	849	853	4	
1068		857	861	865	869	873	877	882	886	890	894	4	
1069		898	902	906	910	914	918	922	926	930	934	4	
1070		938	942	946	951	955	959	963	967	971	975	4	
1071		979	983	987	991	995	999	003	007	011	015	4	
1072	03	019	024	028	032	036	040	044	048	052	056	4	
1073		060	064	068	072	076	080	084	088	092	096	4	
1074		100	104	109	113	117	121	125	129	133	137	4	
1075		141	145	149	153	157	161	165	169	173	177	4	
1076		181	185	189	193	197	201	205	209	214	218	4	
1077		222	226	230	234	238	242	246	250	254	258	4	
1078		262	266	270	274	278	282	286	290	294	298	4	
1079		302	306	310	314	318	322	326	330	334	338	4	
1080		342	346	350	354	358	362	366	371	375	379	4	
1081		383	387	391	395	399	403	407	411	415	419	4	
1082		423	427	431	435	439	443	447	451	455	459	4	
1083		463	467	471	475	479	483	487	491	495	499	4	
1084		503	507	511	515	519	523	527	531	535	539	4	
1085		543	547	551	555	559	563	567	571	575	579	4	
1086		583	587	591	595	599	603	607	611	615	619	4	
1087		623	627	631	635	639	643	647	651	655	659	4	
1088		663	667	671	675	679	683	687	691	695	699	4	
1089		703	707	711	715	719	723	727	731	735	739	4	
1090		743	747	751	755	759	763	767	771	775	778	4	
1091		782	786	790	794	798	802	806	810	814	818	4	
1092		822	826	830	834	838	842	846	850	854	858	4	
1093		862	866	870	874	878	882	886	890	894	898	4	
1094		902	906	910	914	918	922	926	930	933	937	4	
1095		941	945	949	953	957	961	965	969	973	977	4	
1096		981	985	989	993	997	001	005	009	013	017	4	
1097	04	021	025	029	033	036	040	044	048	052	056	4	
1098		060	064	068	072	076	080	084	088	092	096	4	
1099		100	104	108	112	116	120	123	127	131	135	4	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

	5
Δn	Δl
2.0	1
4.0	2
6.0	3
8.0	4
10	5

	4
Δn	Δl
2.5	1
5.0	2
7.5	3
10	4

	3
Δn	Δl
3.3	1
6.7	2
10	3

FIVE-PLACE LOGARITHMS.

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
1100	04	139	143	147	151	155	159	163	167	171	175	4	
1101		179	183	187	191	195	198	202	206	210	214	4	
1102		218	222	226	230	234	238	242	246	250	254	4	
1103		258	261	265	269	273	277	281	285	289	293	4	
1104		297	301	305	309	313	317	321	324	328	332	4	
1105		336	340	344	348	352	356	360	364	368	372	4	
1106		376	379	383	387	391	395	399	403	407	411	4	
1107		415	419	423	427	430	434	438	442	446	450	4	
1108		454	458	462	466	470	474	477	481	485	489	4	
1109		493	497	501	505	509	513	517	521	524	528	4	
1110		532	536	540	544	548	552	556	560	564	567	4	
1111		571	575	579	583	587	591	595	599	603	607	3	
1112		610	614	618	622	626	630	634	638	642	646	4	
1113		650	653	657	661	665	669	673	677	681	685	4	
1114		689	692	696	700	704	708	712	716	720	724	3	
1115		727	731	735	739	743	747	751	755	759	763	3	$\Delta n \begin{array}{l} 4 \\ \hline 1 \end{array}$
1116		766	770	774	778	782	786	790	794	798	801	4	
1117		805	809	813	817	821	825	829	833	836	840	4	
1118		844	848	852	856	860	864	867	871	875	879	4	
1119		883	887	891	895	899	902	906	910	914	918	4	
1120		922	926	930	933	937	941	945	949	953	957	4	
1121		961	964	968	972	976	980	984	988	992	995	4	
1122		999	003	007	011	015	019	023	026	030	034	4	
1123		038	042	046	050	053	057	061	065	069	073	4	
1124		077	080	084	088	092	096	100	104	108	111	4	
1125		115	119	123	127	131	135	138	142	146	150	4	
1126		154	158	162	165	169	173	177	181	185	189	3	
1127		192	196	200	204	208	212	216	219	223	227	4	
1128		231	235	239	242	246	250	254	258	262	266	3	
1129		269	273	277	281	285	289	292	296	300	304	4	
1130		308	312	316	319	323	327	331	335	339	342	4	
1131		346	350	354	358	362	365	369	373	377	381	4	
1132		385	388	392	396	400	404	408	411	415	419	4	
1133		423	427	431	434	438	442	446	450	454	457	4	
1134		461	465	469	473	477	480	484	488	492	496	4	
1135		500	503	507	511	515	519	523	526	530	534	4	$\Delta n \begin{array}{l} 3 \\ \hline 1 \end{array}$
1136		538	542	545	549	553	557	561	565	568	572	4	
1137		576	580	584	588	591	595	599	603	607	610	4	
1138		614	618	622	626	629	633	637	641	645	649	3	
1139		652	656	660	664	668	671	675	679	683	687	3	
1140		690	694	698	702	706	710	713	717	721	725	4	
1141		729	732	736	740	744	748	751	755	759	763	4	
1142		767	770	774	778	782	786	789	793	797	801	4	
1143		805	808	812	816	820	824	827	831	835	839	4	
1144		843	846	850	854	858	862	865	869	873	877	4	
1145		881	884	888	892	896	900	903	907	911	915	3	
1146		918	922	926	930	934	937	941	945	949	953	3	
1147		956	960	964	968	971	975	979	983	987	990	4	
1148		994	998	002	006	009	013	017	021	024	028	4	
1149		032	036	040	043	047	051	055	058	062	066	4	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

FIVE-PLACE LOGARITHMS.

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N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
1150	06	070	074	077	081	085	089	092	096	100	104	4	
1151		108	111	115	119	123	126	130	134	138	141	4	
1152		145	149	153	157	160	164	168	172	175	179	4	
1153		183	187	190	194	198	202	206	209	213	217	4	
1154		221	224	228	232	236	239	243	247	251	254	4	
1155		258	262	266	269	273	277	281	285	288	292	4	
1156		296	300	303	307	311	315	318	322	326	330	3	
1157		333	337	341	345	348	352	356	360	363	367	4	
1158		371	375	378	382	386	390	393	397	401	405	3	
1159		408	412	416	420	423	427	431	435	438	442	4	
1160		446	450	453	457	461	465	468	472	476	479	4	
1161		483	487	491	494	498	502	506	509	513	517	4	
1162		521	524	528	532	536	539	543	547	551	554	4	
1163		558	562	565	569	573	577	580	584	588	592	3	
1164		595	599	603	606	610	614	618	621	625	629	4	
1165		633	636	640	644	648	651	655	659	662	666	4	
1166		670	674	677	681	685	688	692	696	700	703	4	
1167		707	711	715	718	722	726	729	733	737	741	3	
1168		744	748	752	755	759	763	767	770	774	778	3	
1169		781	785	789	793	796	800	804	807	811	815	4	
1170		819	822	826	830	833	837	841	845	848	852	4	
1171		856	859	863	867	871	874	878	882	885	889	4	
1172		893	896	900	904	908	911	915	919	922	926	4	
1173		930	934	937	941	945	948	952	956	959	963	4	
1174		967	971	974	978	982	985	989	993	996	1000	4	
1175	07	004	007	011	015	019	022	026	030	033	037	4	
1176		041	044	048	052	056	059	063	067	070	074	4	
1177		078	081	085	089	092	096	100	103	107	111	4	
1178		115	118	122	126	129	133	137	140	144	148	3	
1179		151	155	159	162	166	170	173	177	181	185	3	
1180		188	192	196	199	203	207	210	214	218	221	4	
1181		225	229	232	236	240	243	247	251	254	258	4	
1182		262	265	269	273	276	280	284	287	291	295	3	
1183		298	302	306	309	313	317	320	324	328	332	3	
1184		335	339	343	346	350	354	357	361	365	368	4	
1185		372	375	379	383	386	390	394	397	401	405	3	
1186		408	412	416	419	423	427	430	434	438	441	4	
1187		445	449	452	456	460	463	467	471	474	478	4	
1188		482	485	489	493	496	500	504	507	511	515	3	
1189		518	522	525	529	533	536	540	544	547	551	4	
1190		555	558	562	566	569	573	577	580	584	588	3	
1191		591	595	598	602	606	609	613	617	620	624	4	
1192		628	631	635	639	642	646	649	653	657	660	4	
1193		664	668	671	675	679	682	686	690	693	697	3	
1194		700	704	708	711	715	719	722	726	730	733	4	
1195		737	740	744	748	751	755	759	762	766	769	4	
1196		773	777	780	784	788	791	795	799	802	806	3	
1197		809	813	817	820	824	828	831	835	838	842	4	
1198		846	849	853	857	860	864	867	871	875	878	4	
1199		882	886	889	893	896	900	904	907	911	915	3	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

Δ n	Δ l
2.5	1
5.0	2
7.5	3
10	4

Δ n	Δ l
3.3	1
6.7	2
10	3

II

III

FIVE-PLACE LOGARITHMS.

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
1200	07	918	922	925	929	933	936	940	943	947	951	3	
1201		954	958	962	965	969	972	976	980	983	987	3	
1202		990	994	998	101	105	109	112	116	119	123	4	
1203	08	027	030	034	037	041	045	048	052	055	059	4	
1204		063	066	070	073	077	081	084	088	091	095	4	
1205		099	102	106	110	113	117	120	124	128	131	4	
1206		135	138	142	146	149	153	156	160	164	167	4	
1207		171	174	178	182	185	189	192	196	200	203	4	
1208		207	210	214	217	221	225	228	232	235	239	4	
1209		243	246	250	253	257	261	264	268	271	275	4	
1210		279	282	286	289	293	296	300	304	307	311	3	
1211		314	318	322	325	329	332	336	340	343	347	3	
1212		350	354	357	361	365	368	372	375	379	382	4	
1213		386	390	393	397	400	404	408	411	415	418	4	
1214		422	425	429	433	436	440	443	447	450	454	4	$\Delta n \Delta l$
1215		458	461	465	468	472	475	479	483	486	490	3	$\frac{4}{2.5}$
1216		493	497	500	504	508	511	515	518	522	525	4	$\frac{1}{5.0}$
1217		529	533	536	540	543	547	550	554	558	561	4	$\frac{2}{7.5}$
1218		565	568	572	575	579	583	586	590	593	597	3	$\frac{3}{10}$
1219		600	604	607	611	615	618	622	625	629	632	4	$\frac{4}{10}$
1220		636	640	643	647	650	654	657	661	664	668	4	
1221		672	675	679	682	686	689	693	696	700	704	3	
1222		707	711	714	718	721	725	728	732	736	739	4	
1223		743	746	750	753	757	760	764	767	771	775	3	
1224		778	782	785	789	792	796	799	803	807	810	4	
1225		814	817	821	824	828	831	835	838	842	846	3	
1226		849	853	856	860	863	867	870	874	877	881	3	
1227		884	888	892	895	899	902	906	909	913	916	4	
1228		920	923	927	930	934	938	941	945	948	952	3	
1229		955	959	962	966	969	973	976	980	983	987	4	
1230		991	994	998	101	105	108	112	115	119	122	4	
1231	09	026	029	033	036	040	043	047	050	054	058	3	
1232		061	065	068	072	075	079	082	086	089	093	3	
1233		096	100	103	107	110	114	117	121	124	128	4	
1234		132	135	139	142	146	149	153	156	160	163	4	$\Delta n \Delta l$
1235		167	170	174	177	181	184	188	191	195	198	4	$\frac{3}{3.3}$
1236		202	205	209	212	216	219	223	226	230	233	4	$\frac{1}{6.7}$
1237		237	240	244	248	251	255	258	262	265	269	3	$\frac{2}{10}$
1238		272	276	279	283	286	290	293	297	300	304	3	
1239		307	311	314	318	321	325	328	332	335	339	3	
1240		342	346	349	353	356	360	363	367	370	374	3	
1241		377	381	384	388	391	395	398	402	405	409	3	
1242		412	416	419	423	426	430	433	437	440	444	3	
1243		447	451	454	458	461	465	468	472	475	479	3	
1244		482	486	489	493	496	499	503	506	510	513	4	
1245		517	520	524	527	531	534	538	541	545	548	4	
1246		552	555	559	562	566	569	573	576	580	583	4	
1247		587	590	594	597	601	604	608	611	614	618	3	
1248		621	625	628	632	635	639	642	646	649	653	3	
1249		656	660	663	667	670	674	677	681	684	688	3	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
1250	09	691	694	698	701	705	708	712	715	719	722	4	
1251		726	729	733	736	740	743	747	750	753	757	3	
1252		760	764	767	771	774	778	781	785	788	792	3	
1253		795	799	802	806	809	812	816	819	823	826	4	
1254		830	833	837	840	844	847	851	854	857	861	3	
1255		864	868	871	875	878	882	885	889	892	896	3	
1256		899	902	906	909	913	916	920	923	927	930	4	
1257		934	937	940	944	947	951	954	958	961	965	3	
1258		968	972	975	978	982	985	989	992	996	999	4	
1259	10	003	006	009	013	016	020	023	027	030	034	3	
1260		037	041	044	047	051	054	058	061	065	068	4	
1261		072	075	078	082	085	089	092	096	099	102	4	
1262		106	109	113	116	120	123	127	130	133	137	3	
1263		140	144	147	151	154	158	161	164	168	171	4	
1264		175	178	182	185	188	192	195	199	202	206	3	
1265		209	212	216	219	223	226	230	233	237	240	3	$\Delta n \Delta 1$
1266		243	247	250	254	257	261	264	267	271	274	4	2.6 1
1267		278	281	285	288	291	295	298	302	305	309	3	5.0 2
1268		312	315	319	322	326	329	332	336	339	343	3	7.6 3
1269		346	350	353	356	360	363	367	370	374	377	3	10 4
1270		380	384	387	391	394	397	401	404	408	411	4	
1271		415	418	421	425	428	432	435	438	442	445	4	
1272		449	452	456	459	462	466	469	473	476	479	4	
1273		483	486	490	493	496	500	503	507	510	514	3	
1274		517	520	524	527	531	534	537	541	544	548	3	
1275		551	554	558	561	565	568	571	575	578	582	3	
1276		585	588	592	595	599	602	605	609	612	616	3	
1277		619	622	626	629	633	636	639	643	646	650	3	
1278		653	656	660	663	667	670	673	677	680	684	3	
1279		687	690	694	697	701	704	707	711	714	718	3	
1280		721	724	728	731	735	738	741	745	748	752	3	
1281		755	758	762	765	768	772	775	779	782	785	4	
1282		789	792	796	799	802	806	809	813	816	819	4	
1283		823	826	829	833	836	840	843	846	850	853	4	
1284		857	860	863	867	870	873	877	880	884	887	3	$\Delta n \Delta 1$
1285		890	894	897	900	904	907	911	914	917	921	3	3.8 1
1286		924	927	931	934	938	941	944	948	951	954	4	6.7 2
1287		958	961	965	968	971	975	978	981	985	988	4	10 3
1288		992	995	998	1002	1005	1008	1012	1015	1019	1022	3	
1289	11	025	029	032	035	039	042	046	049	052	056	3	
1290		059	062	066	069	072	076	079	083	086	089	4	
1291		093	096	099	103	106	109	113	116	120	123	3	
1292		126	130	133	136	140	143	146	150	153	156	4	
1293		160	163	167	170	173	177	180	183	187	190	3	
1294		193	197	200	203	207	210	214	217	220	224	3	
1295		227	230	234	237	240	244	247	250	254	257	4	
1296		261	264	267	271	274	277	281	284	287	291	3	
1297		294	297	301	304	307	311	314	317	321	324	3	
1298		327	331	334	338	341	344	348	351	354	358	3	
1299		361	364	368	371	374	378	381	384	388	391	3	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

FIVE-PLACE LOGARITHMS.

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
1300	11	394	398	401	404	408	411	414	418	421	424	4	
1301		428	431	434	438	441	444	448	451	454	458	3	
1302		461	464	468	471	474	478	481	484	488	491	3	
1303		494	498	501	504	508	511	514	518	521	524	4	
1304		528	531	534	538	541	544	548	551	554	558	3	
1305		561	564	568	571	574	578	581	584	588	591	3	
1306		594	598	601	604	608	611	614	618	621	624	4	
1307		628	631	634	638	641	644	647	651	654	657	4	
1308		661	664	667	671	674	677	681	684	687	691	3	
1309		694	697	701	704	707	711	714	717	720	724	3	
1310		727	730	734	737	740	744	747	750	754	757	3	
1311		760	764	767	770	774	777	780	783	787	790	3	
1312		793	797	800	803	807	810	813	817	820	823	3	
1313		826	830	833	836	840	843	846	850	853	856	4	
1314		860	863	866	869	873	876	879	883	886	889	4	
1315		893	896	899	902	906	909	912	916	919	922	4	$\Delta n \Delta l$ 4
1316		926	929	932	935	939	942	945	949	952	955	4	2.5 1
1317		959	962	965	968	972	975	978	982	985	988	4	5.0 2
1318		992	995	998	101	105	108	111	115	118	121	3	7.5 3
1319	12	024	028	031	034	038	041	044	048	051	054	3	10 4
1320		057	061	064	067	071	074	077	080	084	087	3	
1321		090	094	097	100	103	107	110	113	117	120	3	
1322		123	126	130	133	136	140	143	146	149	153	3	
1323		156	159	163	166	169	172	176	179	182	186	3	
1324		189	192	195	199	202	205	208	212	215	218	4	
1325		222	225	228	231	235	238	241	245	248	251	3	
1326		254	258	261	264	267	271	274	277	281	284	3	
1327		287	290	294	297	300	303	307	310	313	317	3	
1328		320	323	326	330	333	336	339	343	346	349	3	
1329		352	356	359	362	366	369	372	375	379	382	3	
1330		385	388	392	395	398	401	405	408	411	415	3	
1331		418	421	424	428	431	434	437	441	444	447	3	
1332		450	454	457	460	463	467	470	473	476	480	3	
1333		483	486	490	493	496	499	503	506	509	512	4	$\Delta n \Delta l$ 3
1334		516	519	522	525	529	532	535	538	542	545	3	3.3 1
1335		548	551	555	558	561	564	568	571	574	577	4	6.7 2
1336		581	584	587	590	594	597	600	603	607	610	3	10 3
1337		613	616	620	623	626	629	633	636	639	642	4	
1338		646	649	652	655	659	662	665	668	672	675	3	
1339		678	681	685	688	691	694	698	701	704	707	3	
1340		710	714	717	720	723	727	730	733	736	740	3	
1341		743	746	749	753	756	759	762	766	769	772	3	
1342		775	778	782	785	788	791	795	798	801	804	4	
1343		808	811	814	817	821	824	827	830	833	837	3	
1344		840	843	846	850	853	856	859	863	866	869	3	
1345		872	875	879	882	885	888	892	895	898	901	4	
1346		905	908	911	914	917	921	924	927	930	934	3	
1347		937	940	943	946	950	953	956	959	963	966	3	
1348		969	972	975	979	982	985	988	992	995	998	3	
1349	13	001	004	008	011	014	017	021	024	027	030	3	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
1350	13	033	037	040	043	046	049	053	056	059	062	4	
1351		066	069	072	075	078	082	085	088	091	094	4	
1352		098	101	104	107	111	114	117	120	123	127	3	
1353		130	133	136	139	143	146	149	152	155	159	3	
1354		162	165	168	171	175	178	181	184	188	191	3	
1355		194	197	200	204	207	210	213	216	220	223	3	
1356		226	229	232	236	239	242	245	248	252	255	3	
1357		258	261	264	268	271	274	277	280	284	287	3	
1358		290	293	296	300	303	306	309	312	316	319	3	
1359		322	325	328	332	335	338	341	344	348	351	3	
1360		354	357	360	363	367	370	373	376	379	383	3	
1361		386	389	392	395	399	402	405	408	411	415	3	
1362		418	421	424	427	430	434	437	440	443	446	4	
1363		450	453	456	459	462	466	469	472	475	478	3	
1364		481	485	488	491	494	497	501	504	507	510	3	
1365		513	516	520	523	526	529	532	536	539	542	3	
1366		545	548	551	555	558	561	564	567	570	574	3	
1367		577	580	583	586	590	593	596	599	602	605	4	
1368		609	612	615	618	621	624	628	631	634	637	3	
1369		640	644	647	650	653	656	659	663	666	669	3	
1370		672	675	678	682	685	688	691	694	697	701	3	
1371		704	707	710	713	716	720	723	726	729	732	3	
1372		735	739	742	745	748	751	754	758	761	764	3	
1373		767	770	773	777	780	783	786	789	792	796	3	
1374		799	802	805	808	811	814	818	821	824	827	3	
1375		830	833	837	840	843	846	849	852	856	859	3	
1376		862	865	868	871	874	878	881	884	887	890	3	
1377		893	897	900	903	906	909	912	915	919	922	3	
1378		925	928	931	934	938	941	944	947	950	953	3	
1379		956	960	963	966	969	972	975	978	982	985	3	
1380		988	991	994	997	1000	1004	1007	1010	1013	1016	3	
1381	14	019	023	026	029	032	035	038	041	045	048	3	
1382		051	054	057	060	063	067	070	073	076	079	3	
1383		082	085	088	092	095	098	101	104	107	110	4	
1384		114	117	120	123	126	129	132	136	139	142	3	
1385		145	148	151	154	158	161	164	167	170	173	3	
1386		176	179	183	186	189	192	195	198	201	205	3	
1387		208	211	214	217	220	223	226	230	233	236	3	
1388		239	242	245	248	251	255	258	261	264	267	3	
1389		270	273	276	280	283	286	289	292	295	298	3	
1390		301	305	308	311	314	317	320	323	326	330	3	
1391		333	336	339	342	345	348	351	355	358	361	3	
1392		364	367	370	373	376	380	383	386	389	392	3	
1393		395	398	401	404	408	411	414	417	420	423	3	
1394		426	429	433	436	439	442	445	448	451	454	3	
1395		457	461	464	467	470	473	476	479	482	485	4	
1396		489	492	495	498	501	504	507	510	513	517	3	
1397		520	523	526	529	532	535	538	541	545	548	3	
1398		551	554	557	560	563	566	569	572	576	579	3	
1399		582	585	588	591	594	597	600	603	607	610	3	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

$$\frac{\Delta n}{\Delta l}$$

2.5	1
5.0	2
7.5	3
10	4

$$\frac{\Delta n}{\Delta l}$$

3.3	1
6.7	2
10	3

FIVE-PLACE LOGARITHMS.

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
1400	14	613	616	619	622	625	628	631	635	638	641	3	
1401		644	647	650	653	656	659	662	666	669	672	3	
1402		675	678	681	684	687	690	693	696	700	703	3	
1403		706	709	712	715	718	721	724	727	731	734	3	
1404		737	740	743	746	749	752	755	758	761	765	3	
1405		768	771	774	777	780	783	786	789	792	795	4	
1406		799	802	805	808	811	814	817	820	823	826	3	
1407		829	832	836	839	842	845	848	851	854	857	3	
1408		860	863	866	870	873	876	879	882	885	888	3	
1409		891	894	897	900	903	907	910	913	916	919	3	
1410	15	922	925	928	931	934	937	940	943	947	950	3	
1411		953	956	959	962	965	968	971	974	977	980	3	
1412		983	987	990	993	996	999	1002	1005	1008	1011	3	
1413		014	017	020	023	027	030	033	036	039	042	3	
1414		045	048	051	054	057	060	063	066	070	073	3	
1415		076	079	082	085	088	091	094	097	100	103	3	Δn Δl $\frac{2.5}{5.0}$ $\frac{1}{2}$ $\frac{7.5}{10}$ $\frac{3}{4}$
1416		106	109	112	116	119	122	125	128	131	134	3	
1417		137	140	143	146	149	152	155	158	161	165	3	
1418		168	171	174	177	180	183	186	189	192	195	3	
1419		198	201	204	207	210	214	217	220	223	226	3	
1420		229	232	235	238	241	244	247	250	253	256	3	
1421		259	262	266	269	272	275	278	281	284	287	3	
1422		290	293	296	299	302	305	308	311	314	317	3	
1423		320	324	327	330	333	336	339	342	345	348	3	
1424		351	354	357	360	363	366	369	372	375	378	3	
1425		381	385	388	391	394	397	400	403	406	409	3	
1426		412	415	418	421	424	427	430	433	436	439	3	
1427		442	445	448	452	455	458	461	464	467	470	3	
1428		473	476	479	482	485	488	491	494	497	500	3	
1429		503	506	509	512	515	518	521	524	528	531	3	
1430		534	537	540	543	546	549	552	555	558	561	3	
1431		564	567	570	573	576	579	582	585	588	591	3	
1432		594	597	600	603	606	609	612	616	619	622	3	
1433		625	628	631	634	637	640	643	646	649	652	3	
1434		655	658	661	664	667	670	673	676	679	682	3	
1435		685	688	691	694	697	700	703	706	709	712	3	Δn Δl $\frac{3.3}{6.7}$ $\frac{1}{2}$ $\frac{10}{10}$ $\frac{3}{3}$
1436		715	718	721	725	728	731	734	737	740	743	3	
1437		746	749	752	755	758	761	764	767	770	773	3	
1438		776	779	782	785	788	791	794	797	800	803	3	
1439		806	809	812	815	818	821	824	827	830	833	3	
1440		836	839	842	845	848	851	854	857	860	863	3	
1441		866	869	872	875	878	881	884	887	891	894	3	
1442		897	900	903	906	909	912	915	918	921	924	3	
1443		927	930	933	936	939	942	945	948	951	954	3	
1444		957	960	963	966	969	972	975	978	981	984	3	
1445	16	987	990	993	996	999	1002	1005	1008	1011	1014	3	
1446		017	020	023	026	029	032	035	038	041	044	3	
1447		047	050	053	056	059	062	065	068	071	074	3	
1448		077	080	083	086	089	092	095	098	101	104	3	
1449		107	110	113	116	119	122	125	128	131	134	3	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.
1450	16	137	140	143	146	149	152	155	158	161	164	3	$\begin{array}{r l} \Delta n & 3 \\ \hline \Delta l & 1 \\ \hline 3.3 & x \\ 6.7 & 2 \\ 10 & 3 \end{array}$
1451		167	170	173	176	179	182	185	188	191	194	3	
1452		197	200	203	206	209	212	215	218	221	224	3	
1453		227	230	233	236	239	242	244	247	250	253	3	
1454		256	259	262	265	268	271	274	277	280	283	3	
1455		286	289	292	295	298	301	304	307	310	313	3	
1456		316	319	322	325	328	331	334	337	340	343	3	
1457		346	349	352	355	358	361	364	367	370	373	3	
1458		376	379	382	385	388	391	394	397	400	403	3	
1459		406	409	411	414	417	420	423	426	429	432	3	
1460		435	438	441	444	447	450	453	456	459	462	3	
1461		465	468	471	474	477	480	483	486	489	492	3	
1462		495	498	501	504	507	510	513	516	518	521	3	
1463		524	527	530	533	536	539	542	545	548	551	3	
1464		554	557	560	563	566	569	572	575	578	581	3	
1465		584	587	590	593	596	599	602	605	607	610	3	
1466		613	616	619	622	625	628	631	634	637	640	3	
1467		643	646	649	652	655	658	661	664	667	670	3	
1468		673	676	679	681	684	687	690	693	696	699	3	
1469		702	705	708	711	714	717	720	723	726	729	3	
1470		732	735	738	741	744	747	749	752	755	758	3	$\begin{array}{r l} \Delta n & 2 \\ \hline \Delta l & 1 \\ \hline 5.0 & 1 \\ 10 & 2 \end{array}$
1471		761	764	767	770	773	776	779	782	785	788	3	
1472		791	794	797	800	803	806	808	811	814	817	3	
1473		820	823	826	829	832	835	838	841	844	847	3	
1474		850	853	856	859	862	864	867	870	873	876	3	
1475		879	882	885	888	891	894	897	900	903	906	3	
1476		909	912	915	917	920	923	926	929	932	935	3	
1477		938	941	944	947	950	953	956	959	962	965	2	
1478		967	970	973	976	979	982	985	988	991	994	3	
1479		997	100	103	106	109	111	114	117	120	123	3	
1480	17	026	029	032	035	038	041	044	047	050	053	3	
1481		056	058	061	064	067	070	073	076	079	082	3	
1482		085	088	091	094	097	099	102	105	108	111	3	
1483		114	117	120	123	126	129	132	135	138	140	3	
1484		143	146	149	152	155	158	161	164	167	170	3	
1485		173	176	178	181	184	187	190	193	196	199	3	
1486		202	205	208	211	214	216	219	222	225	228	3	
1487		231	234	237	240	243	246	249	252	254	257	3	
1488		260	263	266	269	272	275	278	281	284	287	2	
1489		289	292	295	298	301	304	307	310	313	316	3	
1490		319	322	324	327	330	333	336	339	342	345	3	$\begin{array}{r l} \Delta n & 2 \\ \hline \Delta l & 1 \\ \hline 5.0 & 1 \\ 10 & 2 \end{array}$
1491		348	351	354	357	359	362	365	368	371	374	3	
1492		377	380	383	386	389	391	394	397	400	403	3	
1493		406	409	412	415	418	421	423	426	429	432	3	
1494		435	438	441	444	447	450	452	455	458	461	3	
1495		464	467	470	473	476	479	482	484	487	490	3	
1496		493	496	499	502	505	508	511	513	516	519	3	
1497		522	525	528	531	534	537	540	542	545	548	3	
1498		551	554	557	560	563	566	569	571	574	577	3	
1499		580	583	586	589	592	595	598	600	603	606	3	
N.	Log	0	1	2	3	4	5	6	7	8	9	Δ	P.P.

TABLE GIVING THE DECIMAL-VALUES F OF ALL INTEGER-RATIOS $x:y$ UP TO $F = 0.5$ AND $y = 60$.

F.	$x:y$.	F.	$x:y$.	F.	$x:y$.	F.	$x:y$.
.016 667	1:60	.040 000	1:25	.075 000	3:40	.111 11	1:9
.016 949	1:59	.040 816	2:49	.075 472	4:53	.113 21	6:53
.017 241	1:58	.041 667	1:24	.076 923	1:13	.113 64	5:44
.017 544	1:57	.042 553	2:47	.078 431	4:51	.114 29	4:35
.017 857	1:56	.043 478	1:23	.078 947	3:38	.115 38	3:26
.018 182	1:55	.044 444	2:45	.080 000	2:25	.116 28	5:43
.018 519	1:54	.045 455	1:22	.081 081	3:37	.116 67	7:60
.018 868	1:53	.046 512	2:43	.081 633	4:49	.117 65	2:17
.019 231	1:52	.047 619	1:21	.083 333	1:12	.118 64	7:59
.019 608	1:51	.048 780	2:41	.084 746	5:59	.119 05	5:42
.020 000	1:50	.050 000	1:20	.085 106	4:47	.120 00	3:25
.020 408	1:49	.050 847	3:59	.085 714	3:35	.120 69	7:58
.020 833	1:48	.051 282	2:39	.086 207	5:58	.121 21	4:33
.021 277	1:47	.051 724	3:58	.086 957	2:23	.121 95	5:41
.021 739	1:46	.052 632	1:19	.087 719	5:57	.122 45	6:49
.022 222	1:45	.053 571	3:56	.088 235	3:34	.122 81	7:57
.022 727	1:44	.054 054	2:37	.088 889	4:45	.125 00	1:8
.023 256	1:43	.054 545	3:55	.089 286	5:56	.127 27	7:55
.023 810	1:42	.055 556	1:18	.090 909	1:11	.127 66	6:47
.024 390	1:41	.056 604	3:53	.092 593	5:54	.128 21	5:39
.025 000	1:40	.057 143	2:35	.093 023	4:43	.129 03	4:31
.025 641	1:39	.057 692	3:52	.093 750	3:32	.129 63	7:54
.026 316	1:38	.058 824	1:17	.094 340	5:53	.130 43	3:23
.027 027	1:37	.059 000	3:50	.095 238	2:21	.131 58	5:38
.027 778	1:36	.060 606	2:33	.096 154	5:52	.132 08	7:53
.028 571	1:35	.061 224	3:49	.096 774	3:31	.133 33	2:15
.029 412	1:34	.062 500	1:16	.097 561	4:41	.134 62	7:52
.030 303	1:33	.063 830	3:47	.098 039	5:51	.135 14	5:37
.031 250	1:32	.064 516	2:31	.100 00	1:10	.135 59	8:59
.032 258	1:31	.065 217	3:46	.101 69	6:59	.136 36	3:22
.033 333	1:30	.066 667	1:15	.102 04	5:49	.137 25	7:51
.033 898	2:59	.067 797	4:59	.102 56	4:39	.137 93	4:29
.034 483	1:29	.068 182	3:44	.103 45	3:29	.138 89	5:36
.035 088	2:57	.068 966	2:29	.104 17	5:48	.139 53	6:43
.035 714	1:28	.069 767	3:43	.105 26	2:19	.140 00	7:50
.036 364	2:55	.070 175	4:57	.106 38	5:47	.140 35	8:57
.037 037	1:27	.071 429	1:14	.107 14	3:28	.142 86	1:7
.037 736	2:53	.072 727	4:55	.108 11	4:37	.145 45	8:55
.038 462	1:26	.073 171	3:41	.108 70	5:46	.145 83	7:48
.039 216	2:51	.074 074	2:27	.109 09	6:55	.146 34	6:41

F.	$x:y$.	F.	$x:y$.	F.	$x:y$.	F.	$x:y$.
.147 06	5 : 34	.191 49	9 : 47	.236 36	13 : 55	.282 05	11 : 39
.148 15	4 : 27	.192 31	5 : 26	.236 84	9 : 38	.282 61	13 : 46
.148 94	7 : 47	.192 98	11 : 57	.237 29	14 : 59	.283 02	15 : 53
.150 00	3 : 20	.193 55	6 : 31	.238 10	5 : 21	.285 71	2 : 7
.150 94	8 : 53	.194 44	7 : 36	.239 13	11 : 46	.288 14	17 : 59
.151 52	5 : 33	.195 12	8 : 41	.240 00	6 : 25	.288 46	15 : 52
.152 17	7 : 46	.195 65	9 : 46	.240 74	13 : 54	.288 89	13 : 45
.152 54	9 : 59	.196 08	10 : 51	.241 38	7 : 29	.289 47	11 : 38
.153 85	2 : 13	.196 43	11 : 56	.242 42	8 : 33	.290 32	9 : 31
.155 17	9 : 58	.200 00	1 : 5	.243 24	9 : 37	.290 91	16 : 55
.155 56	7 : 45	.203 39	12 : 59	.243 90	10 : 41	.291 67	7 : 24
.156 25	5 : 32	.203 70	11 : 54	.244 44	11 : 45	.292 68	12 : 41
.156 86	8 : 51	.204 08	10 : 49	.244 90	12 : 49	.293 10	17 : 58
.157 89	3 : 19	.204 55	9 : 44	.245 28	13 : 53	.294 12	5 : 17
.159 09	7 : 44	.205 13	8 : 39	.245 61	14 : 57	.295 45	13 : 44
.160 00	4 : 25	.205 88	7 : 34	.250 00	1 : 4	.296 30	8 : 27
.160 71	9 : 56	.206 90	6 : 29	.254 24	15 : 59	.297 30	11 : 37
.161 29	5 : 31	.207 55	11 : 53	.254 55	14 : 55	.297 87	14 : 47
.162 16	6 : 37	.208 33	5 : 24	.254 90	13 : 51	.298 25	17 : 57
.162 79	7 : 43	.209 30	9 : 43	.255 32	12 : 47	.300 00	3 : 10
.163 27	8 : 49	.210 53	4 : 19	.255 81	11 : 43	.301 89	16 : 53
.163 64	9 : 55	.211 54	11 : 52	.256 41	10 : 39	.302 33	13 : 43
.166 67	1 : 6	.212 12	7 : 33	.257 14	9 : 35	.303 03	10 : 33
.169 49	10 : 59	.212 77	10 : 47	.258 06	8 : 31	.303 57	17 : 56
.169 81	9 : 53	.214 29	3 : 14	.258 62	15 : 58	.304 35	7 : 23
.170 21	8 : 47	.215 69	11 : 51	.259 26	7 : 27	.305 08	18 : 59
.170 73	7 : 41	.216 22	8 : 37	.260 00	13 : 50	.305 56	11 : 36
.171 43	6 : 35	.216 67	13 : 60	.260 87	6 : 23	.306 12	15 : 49
.172 41	5 : 29	.217 39	5 : 23	.261 90	11 : 42	.307 69	4 : 13
.173 08	9 : 52	.218 18	12 : 55	.263 16	5 : 19	.309 09	17 : 55
.173 91	4 : 23	.218 75	7 : 32	.264 15	14 : 53	.309 52	13 : 42
.175 00	7 : 40	.219 51	9 : 41	.264 71	9 : 34	.310 34	9 : 29
.175 44	10 : 57	.220 00	11 : 50	.265 31	13 : 49	.311 11	14 : 45
.176 47	3 : 17	.220 34	13 : 59	.266 67	4 : 15	.312 50	5 : 16
.177 78	8 : 45	.222 22	2 : 9	.267 86	15 : 56	.313 73	16 : 51
.178 57	5 : 28	.224 14	13 : 58	.268 29	11 : 41	.314 29	11 : 35
.179 49	7 : 39	.224 49	11 : 49	.269 23	7 : 26	.314 81	17 : 54
.180 00	9 : 50	.225 00	9 : 40	.270 27	10 : 37	.315 79	6 : 19
.181 82	2 : 11	.225 81	7 : 31	.270 83	13 : 48	.316 67	19 : 60
.183 33	11 : 60	.226 42	12 : 53	.271 19	16 : 59	.317 07	13 : 41
.183 67	9 : 49	.227 27	5 : 22	.272 73	3 : 11	.318 18	7 : 22
.184 21	7 : 38	.228 07	13 : 57	.274 51	14 : 51	.319 15	15 : 47
.185 19	5 : 27	.228 57	8 : 35	.275 00	11 : 40	.320 00	8 : 25
.186 05	8 : 43	.229 17	11 : 48	.275 86	8 : 29	.320 75	17 : 53
.186 44	11 : 59	.230 77	3 : 13	.276 60	13 : 47	.321 43	9 : 28
.187 50	3 : 16	.232 14	13 : 56	.277 78	5 : 18	.322 03	19 : 59
.188 68	10 : 53	.232 56	10 : 43	.279 07	12 : 43	.322 58	10 : 31
.189 19	7 : 37	.233 33	7 : 30	.280 00	7 : 25	.323 53	11 : 34
.189 66	11 : 58	.234 04	11 : 47	.280 70	16 : 57	.324 32	12 : 37
.190 48	4 : 21	.235 29	4 : 17	.281 25	9 : 32	.325 00	13 : 40

F.	$x:y$.	F.	$x:y$.	F.	$x:y$.	F.	$x:y$.
.325 58	14 : 43	.372 88	22 : 59	.419 35	13 : 31	.464 29	13 : 28
.326 09	15 : 46	.375 00	3 : 8	.420 00	21 : 50	.465 12	20 : 43
.326 53	16 : 49	.377 36	20 : 53	.421 05	8 : 19	.465 52	27 : 58
.326 92	17 : 52	.377 78	17 : 45	.422 22	19 : 45	.466 67	7 : 15
.327 27	18 : 55	.378 38	14 : 37	.423 08	11 : 26	.468 09	22 : 47
.327 59	19 : 58	.379 31	11 : 29	.423 73	25 : 59	.468 75	15 : 32
.333 33	1 : 3	.380 00	19 : 50	.424 24	14 : 33	.469 39	23 : 49
.338 98	20 : 59	.380 95	8 : 21	.425 00	17 : 40	.470 59	8 : 17
.339 29	19 : 56	.381 82	21 : 55	.425 53	20 : 47	.471 70	25 : 53
.339 62	18 : 53	.382 35	13 : 34	.425 93	23 : 54	.472 22	17 : 36
.340 00	17 : 50	.382 98	18 : 47	.428 57	3 : 7	.472 73	26 : 55
.340 43	16 : 47	.383 33	23 : 60	.431 03	25 : 58	.473 68	9 : 19
.340 91	15 : 44	.384 62	5 : 13	.431 37	22 : 51	.474 58	28 : 59
.341 46	14 : 41	.385 96	22 : 57	.431 82	19 : 44	.475 00	19 : 40
.342 11	13 : 38	.386 36	17 : 44	.432 43	16 : 37	.476 19	10 : 21
.342 86	12 : 35	.387 10	12 : 31	.433 33	13 : 30	.477 27	21 : 44
.343 75	11 : 32	.387 76	19 : 49	.433 96	23 : 53	.478 26	11 : 23
.344 83	10 : 29	.388 89	7 : 18	.434 78	10 : 23	.479 17	23 : 48
.345 45	19 : 55	.389 83	23 : 59	.435 90	17 : 39	.480 00	12 : 25
.346 15	9 : 26	.390 24	16 : 41	.436 36	24 : 55	.480 77	25 : 52
.346 94	17 : 49	.391 30	9 : 23	.437 50	7 : 16	.481 48	13 : 27
.347 83	8 : 23	.392 16	20 : 51	.438 60	25 : 57	.482 14	27 : 56
.348 84	15 : 43	.392 86	11 : 28	.439 02	18 : 41	.482 76	14 : 29
.350 00	7 : 20	.393 94	13 : 33	.440 00	11 : 25	.483 33	29 : 60
.350 88	20 : 57	.394 74	15 : 38	.440 68	26 : 59	.483 87	15 : 31
.351 35	13 : 37	.395 35	17 : 43	.441 18	15 : 34	.484 85	16 : 33
.351 85	19 : 54	.395 83	19 : 48	.441 86	19 : 43	.485 71	17 : 35
.352 94	6 : 17	.396 23	21 : 53	.442 31	23 : 52	.486 49	18 : 37
.354 17	17 : 48	.396 55	23 : 58	.444 44	4 : 9	.487 18	19 : 39
.354 84	11 : 31	.400 00	2 : 5	.446 43	25 : 56	.487 80	20 : 41
.355 56	16 : 45	.403 51	23 : 57	.446 81	21 : 47	.488 37	21 : 43
.355 93	21 : 59	.403 85	21 : 52	.447 37	17 : 38	.488 89	22 : 45
.357 14	5 : 14	.404 26	19 : 47	.448 28	13 : 29	.489 36	23 : 47
.358 49	19 : 53	.404 76	17 : 42	.448 98	22 : 49	.489 80	24 : 49
.358 97	14 : 39	.405 41	15 : 37	.450 00	9 : 20	.490 20	25 : 51
.360 00	9 : 25	.406 25	13 : 32	.450 98	23 : 51	.490 57	26 : 53
.361 11	13 : 36	.406 78	24 : 59	.451 61	14 : 31	.490 91	27 : 55
.361 70	17 : 47	.407 41	11 : 27	.452 38	19 : 42	.491 23	28 : 57
.362 07	21 : 58	.408 16	20 : 49	.452 83	24 : 53	.491 53	29 : 59
.363 64	4 : 11	.409 09	9 : 22	.454 55	5 : 11	.500 00	1 : 2
.365 38	19 : 52	.410 26	16 : 39	.456 14	26 : 57		
.365 85	15 : 41	.410 71	23 : 56	.456 52	21 : 46		
.366 67	11 : 30	.411 76	7 : 17	.457 14	16 : 35		
.367 35	18 : 49	.413 04	19 : 46	.457 63	27 : 59		
.368 42	7 : 19	.413 79	12 : 29	.458 33	11 : 24		
.369 56	17 : 46	.414 63	17 : 41	.459 46	17 : 37		
.370 37	10 : 27	.415 09	22 : 53	.460 00	23 : 50		
.371 43	13 : 35	.416 67	5 : 12	.461 54	6 : 13		
.372 09	16 : 43	.418 18	23 : 55	.462 96	25 : 54		
.372 55	19 : 51	.418 60	18 : 43	.463 41	19 : 41		

VALUES, P, OF THE "PROBABILITY-INTEGRAL" IN FUNCTION OF x/r , WHERE x IS A STATED ERROR-LIMIT, AND r THE "PROBABLE ERROR."*

Explanatory Example.—Case of 1000 similar determinations. Of the 1000 errors (taken irrespective of $\pm$), 500 do not exceed r units. Then: 54 don't exceed $0.1 r$; 289 don't exceed $0.55 r$; 993 don't exceed $4 r$, &c.

$\frac{x}{r}$	P.	0	1	2	3	4	5	6	7	8	9	Δ
0.0	.000	.005	.011	.016	.022	.027	.032	.038	.043	.048		6
0.1	.054	.059	.065	.070	.075	.081	.086	.091	.097	.102		5
0.2	.107	.113	.118	.123	.129	.134	.139	.145	.150	.155		5
0.3	.160	.166	.171	.176	.181	.187	.192	.197	.202	.208		5
0.4	.213	.218	.223	.228	.233	.239	.244	.249	.254	.259		5
0.5	.264	.269	.274	.279	.284	.289	.294	.299	.304	.309		5
0.6	.314	.319	.324	.329	.334	.339	.344	.349	.354	.358		5
0.7	.363	.368	.373	.378	.382	.387	.392	.397	.401	.406		5
0.8	.411	.415	.420	.424	.429	.434	.438	.443	.447	.452		4
0.9	.456	.461	.465	.470	.474	.478	.483	.487	.491	.496		4
1.0	.500	.504	.509	.513	.517	.521	.525	.530	.534	.538		4
1.1	.542	.546	.550	.554	.558	.562	.566	.570	.574	.578		4
1.2	.582	.586	.589	.593	.597	.601	.605	.608	.612	.616		3
1.3	.619	.623	.627	.630	.634	.638	.641	.645	.648	.652		3
1.4	.655	.658	.662	.665	.669	.672	.675	.679	.682	.685		3
1.5	.688	.692	.695	.698	.701	.704	.707	.710	.713	.717		3
1.6	.720	.723	.726	.728	.731	.734	.737	.740	.743	.746		3
1.7	.749	.751	.754	.757	.759	.762	.765	.768	.770	.773		2
1.8	.775	.778	.780	.783	.785	.788	.790	.793	.795	.798		2
1.9	.800	.802	.805	.807	.809	.812	.814	.816	.818	.821		2
2.0	.823	.825	.827	.829	.831	.833	.835	.837	.839	.841		2
2.1	.843	.845	.847	.849	.851	.853	.855	.857	.859	.860		2
2.2	.862	.864	.866	.867	.869	.871	.873	.874	.876	.878		1
2.3	.879	.881	.882	.884	.886	.887	.889	.890	.892	.893		2
2.4	.895	.896	.897	.899	.900	.902	.903	.904	.906	.907		1
2.5	.908	.910	.911	.912	.913	.915	.916	.917	.918	.919		1
2.6	.921	.922	.923	.924	.925	.926	.927	.928	.929	.930		1
2.7	.931	.932	.933	.934	.935	.936	.937	.938	.939	.940		1
2.8	.941	.942	.943	.944	.945	.945	.946	.947	.948	.949		1
2.9	.950	.950	.951	.952	.953	.953	.954	.955	.956	.956		1
3.	.957	.964	.969	.974	.978	.982	.985	.987	.990	.992		7 to 1
4.	.993	.994	.995	.996	.997	.998	.998	.999	.999	.999		1,, 0
5.	.9993	.9994	.9996	.9997	.9997	.9998	.9998	.9999	.9999	.9999		1,, 0

* Abridged from a table in Merriman's Method of the least Squares, London, Macmillan; where the values P are given to 4 decimals.

FORMULA-VALUES F OF A NUMBER OF SUBSTANCES AND
RADICALS, AND THEIR LOGARITHMS.

The Characteristics of the Logarithms are omitted.

Name or Formula.	F (O=16.)	Log. F.
Aluminium, Al	27·10	.4330
Al_2O_3	102·20	.0095
$\text{Al}_2(\text{SO}_4)_3$ -	342·38	.5345
$\text{Al}_2\text{K}_2(\text{SO}_4)_4 + 24\text{H}_2\text{O}$ -	948·83	.9772
$\text{Al}_2(\text{NH}_4)_2(\text{SO}_4)_4 + 24\text{H}_2\text{O}$ -	906·67	.9574
$\text{Al}_2\text{P}_2\text{O}_8$ -	244·28	.3879
Antimony, Sb -	119·9	.0788
Sb_2O_3	287·8	.4591
Sb_2O_4	303·8	.4826
$(\text{SbO})\text{K}.\text{C}_4\text{H}_4\text{O}_6 + \frac{1}{2}\text{H}_2\text{O}$	332·0	.5211
Sb_2S_3 -	336·0	.5263
Arsenic, As -	75·09	.8756
As_2O_3 -	198·18	.2971
As_2O_5 -	230·18	.3621
As_2S_3 -	246·36	.3916
$\text{AsO}_4\text{Mg}(\text{NH}_4)$	181·52	.2589
$\text{As}_2\text{O}_7\text{Mg}_2$ -	310·92	.4926
$\text{AsO}_4\text{H}_2\text{K}$ -	180·23	.2558
Barium, Ba -	137·20	.13735
BaO -	153·20	.18526
$\text{Ba}(\text{OH})_2$ -	170·20	.23350
$\text{Ba}(\text{OH})_2 + 8\text{H}_2\text{O}$	315·24	.49866
BaCl_2 -	208·11	.31829
$\text{BaCl}_2 + 2\text{H}_2\text{O}$	244·12	.38760
BaN_2O_6 -	261·29	.41712
BaSO_4 -	233·26	.36784
BaCO_3 -	197·20	.29491
Bismuth, Bi -	208·0	.3181
Bi_2O_3 -	464·00	.6665
BiOCl -	259·45	.4141
$\text{Bi}(\text{NO}_3)_3 \cdot 5\text{H}_2\text{O}$ -	484·16	.6850
$\text{BiO}(\text{NO}_3)$ -	286·0	.4564

Name or Formula.	F (O=16.)	Log. F.
Boron, B	10.9	.0374
B ₂ O ₃ -	69.8	.8439
Na ₂ O.B ₄ O ₆ -	201.7	.3047
Na ₂ O.B ₄ O ₆ .10H ₂ O -	381.8	.5818
BF ₄ K -	126.4	.1017
Bromine, Br	79.952	.90283
HBr -	80.95	.90822
AgBr -	187.88	.27388
KBr -	119.09	.07588
KBrO ₃ -	167.09	.22295
Br - Cl -	44.498	.64834
Cadmium, Cd	112.0	.0492
CdO -	128.0	.1072
CdS -	144.06	.1585
CdSO ₄ -	208.06	.3182
Calcium, Ca	40.02	.60228
CaO -	56.02	.74834
CaCO ₃ -	100.02	.00009
CaC ₂ O ₄ -	128.02	.10728
CaC ₂ O ₄ .H ₂ O (110°)	146.02	.16444
CaSO ₄ -	136.08	.13379
CaSO ₄ .2H ₂ O -	172.09	.23576
CaCl ₂ -	110.93	.04505
CaCl ₂ .6H ₂ O -	218.96	.34037
Carbon, C	12.00	.07918

Multiples by n.

n	nC	n	nC	n	nC	n	nC
11	132	21	252	31	372	41	492
12	144	22	264	32	384	42	504
13	156	23	276	33	396	43	516
14	168	24	288	34	408	44	528
15	180	25	300	35	420	45	540
16	192	26	312	36	432	46	552
17	204	27	324	37	444	47	564
18	216	28	336	38	456	48	576
19	228	29	348	39	468	49	588

Name or Formula.	F (O=16.)	Log. F.
CARBON—Continued.		
CO	28·00	.44716
CO ₂	44·00	.64345
C ₂ O ₄ H ₂ (Oxalic Acid)	90·00	.95424
C ₂ O ₄ H ₂ + 2H ₂ O	126·01	.10041
C ₂ O ₄ ·(NH ₄) ₂ + H ₂ O	142·12	.15266
C ₄ H ₆ O ₆ (Tartaric Acid)	150·01	.17612
C ₆ H ₁₀ O ₅ (Cellulose, Starch)	162·02	.20957
½ (C ₁₂ H ₂₂ O ₁₁) (Cane Sugar, Anhydrous Milk Sugar, Maltose)	171·03	.23307
C ₆ H ₁₂ O ₆ (Glucose, Crystallised Milk Sugar)	180·03	.25535
Chlorine, Cl	35·454	.54967
2Cl	70·91	.85071
3Cl	106·36	.02678
4Cl	141·82	.15174
5Cl	177·27	.24864
6Cl	212·72	.32781
7Cl	248·18	.39477
8Cl	283·63	.45275
9Cl	319·09	.50391
HCl	36·46	.56182
Cl ₂ - O	54·91	.73965
AgCl	143·38	.15649
KCl	74·59	.87268
NaCl	58·507	.76721
KClO ₃	122·59	.08846
KClO ₄	138·59	.14173
Cyanogen, NC; see Nitrogen.		
Chromium, Cr	52·13	.7171
Cr ₂ O ₃	152·26	.1826
Cr ₂ O ₇ K ₂	294·53	.4691
CrO ₃	100·13	.0006
Cobalt, Co	58·7	.7686
CoO	74·7	.8733

Name or Formula.	F (O=16.)	Log. F.
COBALT—Continued.		
CoSO_4 - - -	154·8	.1898
$\text{Co}_2\text{P}_2\text{O}_7$ - -	291·5	.4646
Copper, Cu -	63·34	.80168
CuO -	79·34	.89949
Cu_2O	142·68	.15436
Cu_2S	158·74	.20069
CuSO_4 - -	159·40	.20249
$\text{CuSO}_4\cdot 5\text{H}_2\text{O}$	249·42	.39693
Cu.NCS -	121·45	.08440
Fluorine, F - -	19·10	.2810
CaF_2 - - -	78·22	.8933
$\text{AlF}_3\cdot 3\text{NaF}$	210·86	.3240
Gold, Au (Thorpe and Laurie),	197·32	.29517
„ Au (Krüss). Ann. 238, 272, year 1887,	197·11	.29471
Hydrogen, H -	1·0024	.00104

Multiples.

2.0048; 3.0072; 4.0096; 5.0120; 6.0144; 7.0168;
8.0192; 9.0216; 10.0240.

Water, H_2O	18·005	.25539
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Multiples.

n	$n\text{H}_2\text{O}$	Log.	n	$n\text{H}_2\text{O}$	Log.
1	18.005	.25539	16	288.080	.45951
2	36.010	.55642	17	306.085	.48584
3	54.015	.73251	18	324.090	.51067
4	72.020	.85745	19	342.095	.53415
5	90.025	.95436	20	360.100	.55642
6	108.030	.03354	21	378.105	.57761
7	126.035	.10049	22	396.110	.59782
8	144.040	.15848	23	414.115	.61712
9	162.045	.20964	24	432.120	.63560
10	180.050	.25539	25	450.125	.65333
11	198.055	.29679	26	468.130	.67037
12	216.060	.33457	27	486.135	.68676
13	234.065	.36934	28	504.140	.70255
14	252.070	.40152	29	522.145	.71779
15	270.075	.43148	30	540.150	.73251

Name or Formula.	F (O=16.)			Log. F.
Iodine, I	-	-	126.85	- .10329
$n = 2$ $nI = 253.70$	3 380.55	4 507.40	5 634.25	
HI	-		127.85	.10670
AgI			234.78	.37066
AgIO ₃	-	-	282.78	.45145
KI -	-	-	165.99	.22008
CuI	-	-	190.19	.27919
I - Cl-	-	-	91.396	.96093
Iron, Fe	-		56.02	.74834
FeO	-		72.02	.85745
Fe ₂ O ₃			160.04	.20423
FeCl ₂			126.93	.10356
FeCl ₃			162.38	.21053
FeCl ₃ .6H ₂ O (Small Yellow Crystals)			270.41	.43202
FeSO ₄ .7H ₂ O	-		278.12	.44423
Fe(NH ₄) ₂ S ₂ O ₈ .6Aq.	-	-	392.28	.59360
Lead, Pb	-		206.9	.31576
PbO			222.9	.34811
PbO ₂ -	-		238.9	.37822
PbS			239.0	.37840
PbSO ₄	-		303.0	.48144
PbCrO ₄			323.0	.50920
PbCl ₂			277.8	.44373
Pb(NO ₃) ₂			331.0	.51983
Pb(C ₂ H ₃ O ₂) ₂ .3H ₂ O			378.9	.57853
	Li = 6.89.		Li = 7.00.	
	F.	Log. F.	F.	Log. F.
Lithium, Li	6.89	.8382	7.00	.8451
Li ₂ O	29.78	.4739	30.00	.4771
Li ₂ O.3H ₂ O	83.80	.9232	84.02	.9244
Li ₂ CO ₃	73.78	.8679	74.00	.8692
Li ₂ SO ₄	- - 109.84	.0408	110.06	.0416
Li ₃ PO ₄	115.71	.0634	116.04	.0646

Name or Formula.		F (O=16.)	Log. F.
Magnesium, Mg	-	24.37	.3869
MgO	-	40.37	.6061
MgSO ₄	-	120.43	.0807
MgSO ₄ .7H ₂ O	-	246.47	.3918
MgCl ₂ .6H ₂ O	-	203.31	.3082
Mg ₂ P ₂ O ₇	-	222.82	.3480
Mg(NH ₄)PO ₄	-	137.47	.1382
Mg (NH ₄)PO ₄ .6H ₂ O	-	245.50	.3901
Manganese, Mn	-	55.00	.74036
MnO	-	71.00	.85126
Mn ₃ O ₄	-	229.00	.35984
Mn ₂ O ₃	-	158.00	.19866
MnO ₂	-	87.00	.93952
MnS	-	87.06	.93982
MnSO ₄	-	151.06	.17915
MnCl ₂	-	125.91	.10006
MnO ₄ K	-	158.14	.19904
Mercury, Hg	-	200.3	.30168
HgO	-	216.3	.33506
HgCl	-	235.75	.37245
HgCl ₂	-	271.21	.43331
HgS	-	232.36	.36616
Nickel, Ni	-	58.7	.7686
NiO	-	74.7	.8733
NiS	-	90.8	.9581
NiSO ₄	-	154.8	.1898
Nitrogen, N	-	14.046	.14755
n = 2	3	4	5
nN = 28.092	42.138	56.184	70.230
NH ₃	-	17.053	.23180
NH ₄	-	18.056	.25662
(NH ₄) ₂ O	-	52.111	.71694
(NH ₄) ₂ SO ₄	-	132.17	.12113
NH ₄ Cl	-	53.510	.72844

Name or Formula.	F (O = 16.)	Log. F.
NITROGEN—Continued.		
NH_4NO_3 -	80.102	.90364
$\text{PtCl}_6(\text{NH}_4)_2$	444.33	.64771
N_2O_8 -	76.09	.88133
N_2O_5 -	108.09	.03379
HNO_3 -	63.05	.79969
KNO_3	101.18	.00510
NaNO_3	85.10	.92993
AgNO_3	169.98	.23040
NC -	26.046	.41574
NC.H	27.048	.43214
AgNC	133.976	.12703
NCS.K -	97.242	.98785
NCS.NH_4	76.162	.88174
Phosphorus, P		
P_2O_5	142.08	.15253
Ag_3PO_4	418.83	.62204
BiPO_4	303.04	.48144
$\text{P}_2\text{O}_7\text{Mg}_2$	222.82	.34795
$\text{P}_2\text{O}_8\text{Ca}_3$	310.14	.49156
$\text{P}_2\text{O}_8\text{CaH}_4$	234.11	.36942
Platinum, Pt		
PtCl_6	195.5	.29115
PtCl_6K_2	408.22	.61089
PtCl_6K_2	486.49	.68707
PtCl_6Na_2	454.33	.65737
$\text{PtCl}_6(\text{NH}_4)_2$	444.33	.64771
Potassium, K		
2K -	39.136	.59258
2K	78.272	.89361
3K	117.408	.06970
4K -	156.544	.19464
K_2O	94.27	.97437
KHO -	56.14	.74927
K_2CO_3	138.27	.14073
KHCO_3	100.14	.00061
KCl	74.59	.87268

Name or Formula.	F (O=16.)	Log. F.
POTASSIUM — <i>Continued.</i>		
KClO ₃	122·59	.08846
KClO ₄	138·59	.14173
KNO ₃	101·18	.00510
K ₂ SO ₄	174·33	.24137
KH ₂ PO ₄	136·18	.13411
K ₂ C ₂ O ₄ ·H ₂ O (Oxalate)	184·28	.26548
KH.C ₄ H ₄ O ₆ (Bitartrate)	188·15	.27450
K ₄ .Fe(NC) ₆ + 3H ₂ O (Yellow Prussiate)	422·85	.62619
Silicon, Si	28·40	.4533
SiO ₂	60·40	.7810
Silver, Ag	107·93	.03314
2Ag	215·86	.33417
3Ag	323·79	.51026
4Ag	431·72	.63520
Ag ₂ O	231·86	.36523
AgCl	143·38	.15649
AgBr	187·88	.27388
AgI	234·78	.37066
Ag ₂ S	247·92	.39431
Ag ₂ SO ₄	311·92	.49404
AgNO ₃	169·98	.23040
Ag(NC)	133·98	.12704
Ag(NCS)	166·04	.22021
Sodium, Na	23·053	.36273
2Na	46·106	.66376
3Na	69·159	.83985
4Na	92·212	.96479
Na ₂ O	62·11	.79316
NaOH	40·06	.60271
Na ₂ CO ₃	106·11	.02576
Na ₂ CO ₃ + 10H ₂ O	286·16	.45661
NaHCO ₃	84·06	.92454
NaCl	58·507	.76721
NaNO ₃	85·10	.92993
Na ₂ SO ₄	142·17	.15281

Name or Formula.	F (O=16.)	Log. F.										
SODIUM—Continued.												
$\text{Na}_2\text{SO}_4 + 10\text{H}_2\text{O}$	322.22	.50815										
$\text{Na}_2\text{S}_2\text{O}_3 + 5\text{H}_2\text{O}$	248.25	.39489										
$\text{Na}_2\text{HPO}_4 + 12\text{H}_2\text{O}$ -	358.21	.55414										
Na_3PO_4 - -	164.20	.21537										
Strontium, Sr												
SrO - - - -	87.52	.94211										
SrCO_3 - - - -	103.52	.01502										
SrSO_4 - - - -	147.52	.16885										
$\text{Sr}(\text{NO}_3)_2$ - - -	183.58	.26383										
$\text{Sr}(\text{NO}_3)_2$ - - -	211.61	.32554										
Sulphur, S												
<table><tr><td>n = 2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>nS = 64.12</td><td>96.18</td><td>128.24</td><td>160.3</td><td>192.36</td></tr></table>	n = 2	3	4	5	6	nS = 64.12	96.18	128.24	160.3	192.36		
n = 2	3	4	5	6								
nS = 64.12	96.18	128.24	160.3	192.36								
SO_3	32.06	.50596										
SO_4												
SO_4H_2												
$\text{SO}_4\text{H}_2 + \frac{1}{12}\text{H}_2\text{O}$ (Marignac's Acid)												
SO_4Ba -												
Tin, Sn												
SnO - - - -												
SnO_2 - - - -												
SnCl_2												
$\text{SnCl}_2 + 2\text{H}_2\text{O}$ (Crystals, Marignac)												
Uranium, U												
UO_2												
U_3O_8 - - - -												
UO_3 - - - -												
$*(\text{UO}_2)(\text{NO}_3)_2 + 6\text{H}_2\text{O}$												
$*(\text{UO}_2)(\text{C}_2\text{H}_3\text{O}_2)_2 + 4\text{H}_2\text{O}$												
* Corresponding to $\frac{1}{2} \text{P}_2\text{O}_5 = 71.04$ in the process of titration.												
Zinc, Zn												
ZnO												
ZnS - - - -												
ZnSO_4 - - - -												
$\text{ZnSO}_4 + 7\text{H}_2\text{O}$												

ANALYTICAL FACTORS AND THEIR LOGARITHMS.

The Characteristics of the Logarithms are omitted.

Given a		Wanted b		Multiply by f f = b ÷ a Log f
<i>Aluminium.</i>				
Al ₂ O ₃		Al ₂		·5303 .7245
Al ₂ O ₃	- -	Al ₂ (SO ₄) ₃	-	3·350 .5251
<i>Antimony.</i>				
Sb ₂ O ₄	-	2Sb -		·7893 .8973
Sb ₂ O ₄	-	Sb ₂ O ₃		·9473 .9765
Sb ₂ S ₃	- -	Sb ₂ -		·7137 .8535
<i>Arsenic.</i>				
As ₂ S ₃	- -	2As		·6096 .7850
As ₂ S ₃	- -	As ₂ O ₃		·8044 .9055
As ₂ S ₃	- -	As ₂ O ₅		·9343 .9705
As ₂ O ₇ Mg ₂	- -	2As		·4830 .6840
As ₂ O ₇ Mg ₂	-	As ₂ O ₃		·6374 .8044
As ₂ O ₇ Mg ₂		As ₂ O ₅		·7403 .8694
2AsO ₄ MgNH ₄		As ₂ O ₇ Mg ₂		·8564 .9327
<i>Barium.</i>				
BaSO ₄		Ba		·58818 - .76951
BaSO ₄		BaO		·65678 .81742
BaCO ₃		BaO		·77688 .89035
BaO	-	Ba -		·89556 .95209
<i>Bismuth.</i>				
Bi ₂ O ₃		2Bi -		·8965 .9526
BiOCl -		Bi	-	·8017 .9040
2BiOCl	-	Bi ₂ O ₃		·8942 .9514
<i>Bromine.</i>				
AgBr	-	Br		·42554 .62894
AgBr		BrH		·43088 .63435
Br - Cl		Br		1·79675 .25449
Br - Cl		AgBr		4·22227 .62555

Given a	Wanted b	Multiply by f f = b ÷ a Log f	
<i>Calcium.</i>			
CaO	Ca	.71439	.85393
CaSO ₄ -	CaO	.41167	.61455
CaCO ₃ -	CaO	.56009	.74826
<i>Chlorine.</i>			
AgCl	Cl	.24727	.39316
AgCl	ClH	.25426	.40527
2AgCl -	(Cl ₂ - O) -	.19147	.28211
Cl ₂	O	.22564	.35342
Ag	Cl	.32849	.51652
<i>Chromium.</i>			
Cr ₂ O ₃	Cr ₂ O ₇ K ₂	1.9344	.28654
Cr ₂ O ₃	2CrO ₃	1.3152	.11901
Cr ₂ O ₃ -	6Fe -	2.2075	.34391
6Fe	Cr ₂ O ₃	.45299	.65609
Cr ₂ O ₃	6FeSO ₄ ·7H ₂ O	10.960	.03980
<i>Cobalt.</i>			
CoO	Co	.7858	.8953
Co	CoO	1.273	.1047
CoSO ₄ -	Co	.3793	.5790
Co ₂ P ₂ O ₇ -	2Co	.4028	.6051
Co ₂ P ₂ O ₇	2CoO	.5125	.7097
<i>Copper.</i>			
CuO	Cu	.7983	.9022
Cu	CuO	1.2526	.0978
CuNCS	CuO	.6533	.8151
CuNCS	Cu	.5215	.7173
Cu ₂ S	2CuO	.9996	.9998
Cu ₂ S	2Cu -	.7980	.9020
<i>Cyanogen.</i>			
Ag -	NC -	.24132	.38260
Ag	NCH	.25061	.39899

Given a	Wanted b	Multiply by f $f = b \div a$ Log f	
<i>Cyanogen—Continued.</i>			
NCAg - -	NC - -	·19441 -	·28871
2NCH -	AgNO ₃ - -	3·1422 -	·49723
<i>Iodine.</i>			
AgI - - -	I - - -	·54029	·73263
I - Cl - -	I - - -	1·3879	·14236
I - Cl - -	AgI - -	2·5688	·40973
CuI - - -	I - - -	·66696	·82410
<i>Iron.</i>			
Fe ₂ O ₃ -	2Fe - -	·70007	·84514
Fe ₂ O ₃	2FeO	·90002	·95425
Fe	FeSO ₄ ·7H ₂ O	4·9647	·69589
Fe ₂ O ₃ -	2FeSO ₄ ·7H ₂ O	3·4756	·54103
FeO	Fe	·77784	·89089
Fe - - -	FeO	1·28561	·10911
2Fe - - -	O - - -	·14281	·15475
<i>Lead.</i>			
PbO - - -	Pb - -	·92822	·96765
Pb - - -	PbO	1·07733	·03235
PbSO ₄ -	PbO	·73574	·86673
PbSO ₄ -	Pb	·68293	·83438
PbCl ₂	Pb	·74476	·87202
PbS	PbO	·93279	·96979
PbS - -	Pb	·86583	·93744
<i>Lithium (if Li = 6.89)</i>			
Li ₂ SO ₄	Li ₂ O -	·2711	·4332
Li ₃ PO ₄	1.5Li ₂ O - -	·3861	·5866
Li ₂ O	2Li - - -	·4627	·6653
<i>Lithium (if Li = 7.00)</i>			
Li ₂ SO ₄	Li ₂ O - -	·2726	·4355
Li ₃ PO ₄	1.5Li ₂ O -	·3878	·5886
Li ₂ O	2Li - - -	·4667	·6690

Given a		Wanted b	Multiply by f f = b ÷ a Log f	
<i>Magnesium.</i>				
MgO	-	Mg -	·6037	·7808
Mg	-	MgO	1·6565	·2192
MgSO ₄	- -	MgO	·3352	·5253
P ₂ O ₅ ·2MgO	-	2MgO	·3624	·5591
<i>Manganese.</i>				
MnO	-	Mn - - -	·7746	·8891
Mn	-	MnO	1·2909	·1109
Mn ₃ O ₄	- -	3MnO	·9301	·9685
MnSO ₄	-	MnO	·4700	·6721
MnS	-	MnO	·8155	·9114
MnO	-	O	·2254	·3529
O		MnO	4·438	·6471
2CO ₂	-	MnO ₂ -	·9886	·9950
2CO ₂		O -	·1818	·2596
<i>Nickel.</i>				
NiO	-	Ni - - -	·7858	·8953
Ni	- -	NiO	1·273	·1047
NiSO ₄	- - -	Ni -	·3793	·5790
<i>Nitrogen.</i>				
N	- - -	NH ₃ -	1·21410	·08425
NH ₃	- -	N -	·82366	·91575
2NH ₃	- -	(NH ₄) ₂ O	1·52790	·18410
(NH ₄) ₂ O	-	2NH ₃	·65449	·81591
NH ₄ Cl	-	NH ₃ -	·31869	·50337
2NH ₄ Cl	-	(NH ₄) ₂ O	·48693	·68747
2NH ₃	-	(NH ₄) ₂ SO ₄	3·87526	·58830
SO ₃	- -	2NH ₃	·42601	·62942
(NH ₄) ₂ SO ₄		2NH ₃	·25805	·41170
PtCl ₆ (NH ₄) ₂		2N	·063223 -	·80087
PtCl ₆ (NH ₄) ₂		2NH ₃	·076758	·88512
PtCl ₆ (NH ₄) ₂	-	(NH ₄) ₂ O -	·11728	·06922

Given a	Wanted b	Multiply by f f = b ÷ a Log f	
<i>Nitrogen—Continued.</i>			
Pt	2N	·14369	·15744
Pt	2NH ₃	·17446	·24169
2NH ₃	N ₂ O ₅	3·1693	·50096
N ₂ O ₅	2NH ₃	·31553	·49904
N ₂ O ₅	2NO ₃ H	1·16657	·06691
2HNO ₃ -	N ₂ O ₅	·85722	·93309
<i>Oxygen.</i>			
2CO ₂	O	·1818	·2596
Mn ₂ O ₇ K ₂ O	5 × O	·2529	·4030
5 × O -	Mn ₂ O ₇ K ₂ O	3·954 -	·5970
Mn ₂ O ₇ K ₂ O	3 × O	·1518	·1812
3 × O	Mn ₂ O ₇ K ₂ O	6·589	·8188
<i>Phosphorus.</i>			
P ₂ O ₅	2P	·43694	·64042
P ₂ O ₇ Mg ₂	2P	·2786	·4450
P ₂ O ₇ Mg ₂	P ₂ O ₅	·6376	·8046
P ₂ O ₈ Ca ₃	P ₂ O ₅	·4581	·6610
P ₂ O ₅	P ₂ O ₈ Ca ₃	2·1829	·3390
P ₂ O ₅	P ₂ O ₈ Ca.H ₄	1·6477	·2169
Phosphomolybdate of Ammonia	} Phosph. Pentoxide,	·03794	·5791
Phosphomolybdate of Ammonia		} Elementary Phosphorus,	·01658
<i>Platinum.</i>			
PtCl ₆ (NH ₄) ₂	Pt	·43998	·64344
Pt	PtCl ₆ (NH ₄) ₂ -	2·2728	·35656
PtCl ₆ K ₂	Pt -	·40185	·60407
Pt - -	PtCl ₆ K ₂ - -	2·4885	·39593
<i>Potassium.</i>			
PtCl ₆ K ₂ - -	2K -	·16089	·20653
PtCl ₆ K ₂	K ₂ O -	·19378	·28730

Given a			Wanted b	Multiply by f f = b ÷ a Log f	
<i>Potassium—Continued.</i>					
PtCl ₆ K ₂			2KCl	·30664	·48663
Pt			K ₂ O	·48221	·68324
Pt			2KCl	·76307	·88256
2KCl	-	-	K ₂ O	·63193	·80067
KCl	-	-	K	·52468	·71990
K ₂ SO ₄	-	-	K ₂ O	·54076	·73301
K ₂ SO ₄	-	°	2K	·44898	·65223
K ₂ O		-	2K	·83028	·91922
2K	-	-	K ₂ O	1·20442	·08078
K ₂ O	-		K ₂ CO ₃	1·4667	·16635
K ₂ CO ₃	-	-	K ₂ O	·68179	·83365
<i>Silver.</i>					
AgCl	-	-	Ag	·75273	·87664
Ag	-	-	AgCl	1·32849	·12336
2Ag	-	-	Ag ₂ O	1·07412	·03105
<i>Sodium.</i>					
2NaCl	-	-	Na ₂ O	·53076	·72490
NaCl	-	-	Na	·39402	·59552
Na ₂ SO ₄			Na ₂ O	·43686	·64034
Na ₂ SO ₄	-	-	2Na	·32431	·51096
Na ₂ O	-	-	2Na	·74238	·87062
2Na	-	-	Na ₂ O	1·34703	·12938
Na ₂ O	-	-	Na ₂ CO ₃	1·70846	·23261
Na ₂ CO ₃		-	Na ₂ O	·58532	·76739
Na ₂ CO ₃	-	-	2Na	·43453	·63802
<i>Sulphur.</i>					
BaSO ₄	-	-	S	·13744	·13812
BaSO ₄	-	-	SO ₃	·34322	·53558
BaSO ₄	-		SO ₄	·41182	·61470
BaSO ₄	-		H ₂ SO ₄	·42041	·62367
SO ₃			H ₂ SO ₄	1·2249	·08810
H ₂ SO ₄			SO ₃	·81640	·91190

Given a	Wanted b	Multiply by f f = b ÷ a Log f	
<i>Tin.</i>			
SnO ₂	Sn	·7861	·8955
SnO ₂	SnO	·8931	·9509
<i>Zinc.</i>			
ZnO	Zn -	·8034	·9049
Zn	ZnO -	1·2447	·0951
ZnS	- ZnO	·8352	·9218
ZnS	Zn	·6710	·8267

EMPIRICAL FACTORS.

I.—FACTORS IN CONNECTION WITH THE CHLOROPLATINATE METHOD FOR DETERMINING POTASSIUM AND AMMONIUM, ACCORDING TO W. DITTMAR AND JOHN M'ARTHUR.*

(a) *Finkener's Method as modified by W. D. and J. M'A.*

Given.	Wanted.	f.	Log. f.
Pt	2KCl	·76084	·88129
Pt -	K ₂ O	·48080	·68196
Pt - - -	2K	·39920	·60119

(b) *Tatlock's Method.*

PtCl ₆ K ₂	- 2KCl	·30627	·48610
Pt	2KCl**	·76016	·88090
PtCl ₆ K ₂	K ₂ O	·19354	·28678
PtCl ₆ K ₂	2K	·16069	·20600

** Refers to the KCl in the Chloroplatinate-precipitate. In all other cases under (a) and (b) and (c) the factor gives the KCl, etc., in the substance analysed.

(c) *Determination of Ammonia given as Sal-Ammoniac.*

PtCl ₆ (NH ₄) ₂	2NH ₄ Cl	·2389	·37822
PtCl ₆ (NH ₄) ₂	2NH ₃	·07614	·88159
PtCl ₆ (NH ₄) ₂	2N	·06271	·79733
Pt	2NH ₄ Cl	·5459	·73711
Pt	2NH ₃	·17397	·24048
Pt	2N	·14331	·15626

* Trans. R.S. Ed.; Vol. 33, Part 2, p. 633.

II.—DETERMINATION OF SUGARS BY MEANS OF FEHLING'S SOLUTION.

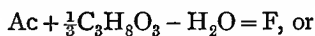
General Modus Operandi.—The sugar is brought into the form of a $\frac{1}{2}$ to 1 per cent. solution. This is poured into *un*-diluted boiling Fehling's Solution, and the boiling continued for, in general, two minutes; in the case of milk sugar for six minutes. Supposing now the copper to be just completely precipitated; then, according to Soxhlet, the following relations hold:

Name of Sugar.	CC. of Fehling per 1 Gramme of Sugar.	Sugar per 1 CC. of Fehling, in Milligrammes.	Grammes of Sugar per 1 Gramme of Metallic Copper in the precipitate.
Dextrose,	210.4	4.753	.5403
Lævulose,	199.4	5.144	.5848
Invert-Sugar,	202.4	4.941	.5617
Galactose,	195.7	5.110	.5809
Milk Sugar, calcu- lated as	148.0	6.756	.7680
$C_{12}H_{24}O_{12}$			
Maltose, $C_{12}H_{22}O_{11}$,	128.5	7.780	.8844

In exact analysis these factors must be accepted only as first approximations, and the exact factors be determined *ex temp.* with known weights of the respective sugars, under the precise conditions of the analysis. (See Dittmar's Exercises in Quantitative Analysis, p. 258.)

EQUIVALENTS OF SOME FATTY ACIDS AND FATS.

If one molecule of fatty acid = Ac, and one molecule of the corresponding fat = 3 F, then:



$$Ac + 12.67 = F.$$

Acid.	Formula.	Ac.	F.
Butyric	$C_4H_8O_2$	88.02	100.69
Caproic -	$C_6H_{12}O_2$	116.03	128.70
Caprylic -	$C_8H_{16}O_2$	144.04	156.71
Capric -	$C_{10}H_{20}O_2$	172.05	184.72
Lauric -	$C_{12}H_{24}O_2$	200.06	212.73
Palmitic -	$C_{16}H_{32}O_2$	256.08	268.75
Linoleic -	$C_{18}H_{34}O_2$	252.07	264.74
Stearic	$C_{18}H_{36}O_2$	284.09	296.76
Oleic	$C_{18}H_{34}O_2$	282.08	294.75
Ricinoleic	$C_{18}H_{34}O_3$	298.08	310.75

GAS VOLUMETRIC DETERMINATIONS.

The Gas obtained is supposed to be measured in cc. (Milli-litres), at $t^{\circ}\text{C}$ and a *dry* pressure of P Millimeters of Mercury of 0°C . "*Weight*" means weight in Milligrammes.

NITROGEN.

Given.	Wanted.	Multiply by.	Log. of Constant.*
Volume of Nitrogen.	Weight of Nitrogen.	$\frac{.45089^* \times P}{272.63 + t}$	.65407 - 1
	Weight of Ammonia, NH_3 .	$\frac{.54743^* \times P}{272.63 + t}$	.73833 - 1
	Weight of Nitric Acid, N_2O_5 .	$\frac{1.7349^* \times P}{272.63 + t}$	.23929
	Weight of Urea, § CON_2H_4 .	$\frac{.9647^* \times P}{272.6 + t}$	.9844 - 1

§ In the ordinary hypobromite process for the determination of Urea in Urine, only 91 per cent. of the nitrogen of the Urea are set free. Our factor applies to the Nitrogen-volume *as corrected* by multiplication with $100 \div 91 = 1.099$.

CARBONIC ACID.

Given.	Wanted.	Multiply by.	Log. of Constant.*
Volume of Carbonic Acid,	Weight of Carbonic Acid, CO_2 .	$\frac{.70174^* \times P}{269.54 + t}$	.84618 - 1

NITRIC ACID.

Given.	Wanted.	Multiply by.	Log. of Constant.*
Volume of Nitric Oxide.	Weight of Nitrous Acid, N_2O_3 .	$\frac{.61102^* \times P}{273 + t}$	.78606 - 1
	Weight of Nitric Acid, N_2O_5 .	$\frac{.86798^* \times P}{273 + t}$	.93851 - 1
	Weight of Ammonia, NH_3 .	$\frac{.27387^* \times P}{273 + t}$	.43755
	Weight of Radical NO_2 .	$\frac{.73950^* \times P}{273 + t}$	.86894 - 1
	Weight of Radical NO_3 .	$\frac{.99646^* \times P}{273 + t}$	.99846 - 1

OXYGEN obtained in LUNGE'S METHOD FOR THE ANALYSIS OF HYPOCHLORITES.

Given.	Wanted.	Multiply by.	Log. of Constant.*
Volume of Oxygen obtained.	Weight of Oxygen in Hypochlorite.	$\frac{.25619^* \times P}{272.18 + t}$	.40857 - 1
	Weight of Active Chlorine.	$\frac{1.1354^* \times P}{272.18 + t}$	.05514

THE METRIC AND THE BRITISH SYSTEMS OF UNITS.

THE METRIC SYSTEM.

Its Fundamental Units are : The Metre, the Litre, and the Gramme. Original Definitions : 1 Metre = the ten-millionth part of the distance from the North Pole to the Equator, measured along the Meridian of Paris. 1 Litre = 1 Cubic-Decimetre. 1000 Grammes = 1 Kilogramme = Mass of 1 Cubic-Decimetre of Water of $+4^{\circ}\text{C}$.

The present *de-facto* definitions are as follows :—The “Metre” is the Length, at 0°C , of a bar of Iridio-Platinum, kept in Paris as the “*Mètre Prototype*.” The “Kilogramme” is the Mass of a block of Iridio-Platinum, preserved in Paris as the “*Kilogramme Prototype*.” The “Litre” is the Volume at $+4^{\circ}\text{C}$, of a quantity of Water, which, in *Vacuo*, balances a true Kilogramme.

The original standard metre was made after the “*Toise du Perou*,” and its length, at 0°C , adjusted to 0.513 074 07* of the length of this Toise at its standard temperature. $\text{Log.}^* = .710\ 180\ 07 - 1$. Hence 1 Toise = 1.949 036 31 Metres. $\text{Log.} = .289\ 819\ 93$. According to Bessel’s calculation the length of the quadrant of a meridian is 10 000 855.76 Metres as thus defined. $\text{Log.} = 7.000\ 037\ 16$.

The *Subsidiary Units* are derived from the fundamental by Multiplication or Division with 10, 100, 1000, &c. Their names are formed by combining those of the fundamental Units with the following prefixes :—Deci- for 0.1 of, Centi- for 0.01 of, Milli- for 0.001 of ; Deka- for 10 times, Hecto- for 100 times, Kilo- for 1000 times, Myria- for 10000 times. Thus, for instance, “Millimetre” means 0.001 Metre ; “Hectolitre” means 100 Litres, &c.

APPENDIX.

For the measurement of Liquids a more convenient unit than the Cubic-Centimetre or Litre is

The “*Fluidgramme*,” which term was introduced by the Author long ago* as designating the volume, at 15°C , of a quantum of Water, the uncorrected weight of which, as determined in Air of 15°C and 760mm pressure by means of brass weights, is one Gramme. Professor de Koninck of Liège has adopted the Author’s unit, but proposed to use the name of “*Mohr*” for 1000 Fluidgrammes, and the name of “*Milli-Mohr*” for the Fluidgramme itself. What, in commercial volumetric apparatus figures as “Cubic Centimetre” or “Litre” comes, as a rule, nearer to our Fluidgramme and Mohr respectively than to the ostensible units.

THE BRITISH UNITS.

Of Length.—The Fundamental Unit is the “Yard,” defined by reference to a material Standard. 1 Mile = 1760 Yards ; 1 Yard = 3 feet = 36 Inches.

* Article : Analyse volumetrische für Flüssigkeiten, etc., in Fehling’s Handwörterbuch der Chemie.

Of Volume.—The Fundamental Unit is the “Gallon” defined as being the Volume, at 62° Fahrenheit of the quantity of Water which, in Air of 62° F. and 30 inch pressure, balances 10 true Pounds Avoirdupois of Brass.

Of Mass.—Fundamental (nominal) Unit: The “Grain” defined by reference to a block of Platinum preserved in London as representing the Weight in *Vacuo* (i.e., the Mass) of “one Pound Avoirdupois” or 7000 Grains.

There are three sets of Units of Weight, viz.:—

- (1) *The Avoirdupois Set.* 1 Ton = 20 Hundredweights (Cwt.) = 2240 Pounds (Lb.) = $16 \times 2240 = 36\,840$ Ounces (Oz.)
1 Lb., by definition, = 7000 Grains, hence 1 Ounce = 437.5 Grains.
- (2) *The Troy Set.* 5760 Grains = 1 Pound = 12 Ounces.
1 Ounce = 20 Penny-weights = $20 \times 24 = 480$ Grains.
- (3) *Apothecaries Weight.* In former times the Troy Ounce was used in Medicine and Pharmacy as unit of Weight. It was divided into 8 Drachmas = $8 \times 3 = 24$ Scruples = $24 \times 20 = 480$ Grains. The present Units are: The Avoirdupois Ounce (= 437.5 Grains) and the Grain. It is, however, permitted to use the names and symbols of “Drachma” and “Scruple” in their old sense, i.e., as designating 60 Grains and 20 Grains respectively.

Note that there is only one kind of Grain in the British System. The words “Pound” and “Ounce,” however, have each two different meanings.

APOTHECARIES FLUID MEASURES.

To define these let us introduce the term “*Fluid-Grain*” as meaning the volume of 1 Grain of Water at 62°F., weighed out, without correction, in Air of this temperature and 30 inch pressure.

1 Fluid-Ounce = 437.5 Fluid-Grains = 8 Fluid-Drachmas = $8 \times 60 = 480$ Minims. Hence 1 Fluid-Drachma = 54.6875 Fluid-Grains; and 1 Minim = 0.911458 Fluid-Grains.

RELATIONS OF BRITISH UNITS TO ONE ANOTHER.

- | | |
|---|----------------------|
| 1 Ton = 32 666.7 Ounces Troy. | Log. = 4.514 105. |
| 1 Pound Avoirdupois = 14 583 3 Ounces Troy. | Log. = 1.163 857. |
| 1 Ounce Avoirdupois = 911 458 Ounces Troy. | Log. = .959 737 - 1. |
| 1 Ounce Troy = 1.097 14 Ounces Avoirdupois. | Log. = .040 263. |
| 1 Gallon, <i>by definition</i> , = 10 Fluid-Pounds = 160 Fluid-Ounces = 70000 Fluid-Grains. <i>By experiment</i> , 1 Gallon = 277.274 Cubic Inches. | |

In Air of 62°F. and 30 inch pressure.

1 Cubic Foot of Water weighs 997.71 Ounces Avoirdupois.

Log. = 3.999 004.

1 Cubic Inch of Water weighs 252.458 Grains. Log. = 2.402 189.

OTHER RELATIONS.

1 Metre = 1.093 633 Yards,	Log. = .038 872.
1 Metre = 3.280 899 Feet.	Log. = .515 993.
1 Metre = 39.370 79 Inches.	Log. = 1.595 173.
1 Yard = .914 383 5 Metres.	Log. = .961 128 - 1.
1 Foot = .304 794 5 Metres.	Log. = .484 007 - 1.
1 Inch = 25.399 54 Millimetres.	Log. = 1.404 827.
1 Cubic Decimetre = 61.027 05 Cub. Inch.	Log. = 1.785 522.
1 Cubic Metre = 35.316 58 Cubic Feet.	Log. = 1.547 979.
1 Cubic Inch = 16.386 18 Cubic Centimetres.	Log. = 1.214 478.
1 Cubic Foot = 28.315 31 Cubic Decim.	Log. = 1.452 022.
1 Litre = .998 081 "Mohr."	Log. = .999 166 - 1.
1 Mohr = 1.001 923 Litres.	Log. = .000 834.
*1 Litre = .219 981 2 Gallons.	Log. = .342 386 - 1.
*1 Litre = 35.197 00 Fluid Ounces.	Log. = 1.546 506.
*1 "Mohr" = 35.264 7 Fluid Ounces.	Log. = 1.547 340.
*1 Gallon = 4.545 85 Litres.	Log. = .657 615.
*1 Fluid-Ounce = 28.357 0 Fluid Gramme.	Log. = 1.452 660.
1 Kilogramme = 2.204 621 Pounds Avoirdupois.	Log. = .343 334.
1 Kilogramme = 35.273 94 Ounces Avoirdupois.	Log. = 1.547 454.
1 Kilogramme = 32.150 7 Ounces Troy.	Log. = 1.507 190.
1 Gramme = 15.432 35 Grains.	Log. = 1.188 432.
1 Gramme = .032 150 7 Ounces Troy.	Log. = .507 190 - 2.
1 Pound Avoirdupois = 453.593 Grammes.	Log. = 2.656 666.
1 Ounce Avoirdupois = 28.349 54 Grammes.	Log. = 1.452 546.
1 Ounce Troy = 31.103 50 Grammes.	Log. = 1.492 810.
1 Ton = 1 016.05 Kilogrammes.	Log. = 3.006 915.

* Calculated by the Author from the original definition of the "Imperial Gallon," the details of which were kindly furnished by Mr. H. J. Chaney, of the Standard Office. One Gallon, by definition, is the volume at 62°F of a quantum of water, which balances ten brass pound-weights (true in vacuo), in air of 62°F and 30 inch (red. to 0°C) pressure, and two-thirds saturated with moisture. The Spec. Gravity of the brass is supposed to be 8.143. By Act of Parliament, however, 1 Gallon is equal to 4.543458 Litres. This number is based upon the experimentally ascertained weight of the cubic inch in grains, and the relation of the cubic *decimetre* to the cubic inch.

CONVERSION OF STATEMENTS OF COMPOSITION.

Given.	Wanted.	Multiply by f . $f =$	Log. $f =$
Milligrammes per Mohr.	Grains per Gallon.	$\cdot 07$	$\cdot 845\ 098 - 2$
Grains per Gallon.	Milligrammes per Mohr.	$14\cdot 286$ $= 100 \div 7$	$1\cdot 154\ 902$
Milligrammes per Litre.	Grains per Gallon.	$\cdot 070\ 135$	$\cdot 845\ 932 - 2$
Grains per Gallon.	Milligrammes per Litre.	$14\cdot 258\ 3$	$1\cdot 154\ 068$
Ounces Troy per Ton.	Milligrammes per Kilo.	$30\cdot 612\ 2$	$1\cdot 485\ 895$
Milligrammes per Kilo.	Ounces Troy per Ton.	$\cdot 032\ 666\ 7$	$\cdot 514\ 105 - 2$

APPENDIX.

VALUES OF GOLD COINS ACCORDING TO THE WEIGHT OF GOLD
CONTAINED IN THEM BY LAW.*

* Calculated after data given in Bremiker's 5-place Logarithm Table, edited by A. Kallius, 1887.

1 Mark =	·979 030	Shilling.	Log. = .990 796 - 1.
1 Shilling =	1·021 42	Marks.	Log. = .009 204.
1 Mark =	048 951 5	Pound.	Log. = .689 766 - 2.
1 Pound =	20·428 4	Marks.	Log. = 1·310 234.
1 Dollar =	4·109 80	Shillings.	Log. = .613 821.
1 Shilling =	·243 321	Dollars.	Log. = .386 179 - 1.
1 Dollar =	·205 490	Pound.	Log. = .312 791 - 1.
1 Pound =	4·866 42	Dollars.	Log. = .687 209.
1 Franc =	·793 014	Shilling.	Log. = .899 281.
1 Shilling =	1·261 01	Francs.	Log. = .100 719.
1 Franc =	·039 650 7	Pound.	Log. = .598 251 - 2.
1 Pound =	25·220 2	Francs.	Log. = 1·401 749.

* The corresponding factors for commercial purposes are liable to fluctuations; hence the factors in the following table are given to only 4 figures; the Logarithms given, as far as they go, are in exact accordance with the data of the first table.

CONVERSION OF PRICES.

Given.	Wanted.	Multiply by f.	
		f =	Log. f =
Price of Pound in Shillings.	Price of Kilogramme in Marks.	2·252	·352 54
Price of Kilogramme in Marks.	Price of Pound in Shillings.	0·444 1	·647 46 - 1
Price of Ounce in Pence.	Price of 100 Grammes in Marks.	·300 2	·477 48 - 1

CONVERSION OF PRICES—*Continued.*

Given.	Wanted.	Multiply by f. f= Log. f=	
Price of 100 Grammes in Marks.	Price of Ounce in Pence.	3.331	.522 51
Price of Pound in Shillings.	Price of Kilogramme in Francs.	2.780	.444 05
Price of Kilogramme in Francs.	Price of Pound in Shillings.	.359 7	.555 95 - 1
Price of Ounce in Pence.	Price of 100 Grammes in Francs.	.370 7	.568 99 - 1
Price of 100 Grammes in Francs.	Price of Ounce in Pence.	2.698	.431 01

VALUE OF PENCE IN DECIMALS OF THE SHILLING.

Pence.	Shilling.	Pence.	Shilling.	Pence.	Shilling.	Pence.	Shilling.
$\frac{1}{2}$	.04	$3\frac{1}{2}$	.29	$6\frac{1}{2}$	.54	$9\frac{1}{2}$	.79
1	.08	4	.33	7	.58	10	.83
$1\frac{1}{2}$	.12	$4\frac{1}{2}$	.37	$7\frac{1}{2}$	.62	$10\frac{1}{2}$	.88
2	.17	5	.42	8	.67	11	.92
$2\frac{1}{2}$	.21	$5\frac{1}{2}$	.46	$8\frac{1}{2}$	.71	$11\frac{1}{2}$	.96
3	.25	6	.50	9	.75	12	1.00

For the conversion of a sum stated in £ s. d. into Pounds and Decimals of the Pound, proceed as shewn in the following example:

$$\text{£}3 \text{ 17s. 9d.} = \text{£}3 + 8 \text{ Florins} + 1 \text{ Shilling} + 36 \text{ Farthings} = \\ \text{£}(3 + 0.8 + 0.05 + .036) \text{£}3.886.$$

(1 Farthing being very nearly = 0.001 Pound) and, conversely, £3.886 reads £3 + (8.5 Florin = 17/) + (36 Farthings = 9d.) = £3 17s. 9d.

HYDROMETER SCALES.

(n = Hydrometer reading ; S = Specific Gravity.)

Hydrometer after	Liquids heavier than Water.	Liquids lighter than Water.
(1) Twaddle ; 60° F.	$S = \frac{n + 200}{200}$ or $n = 200(S - 1)$	Does not exist.
*(2) Gay-Lussac, Centigrade ; 4° C §	$S = \frac{100}{100 - n}$	$S = \frac{100}{100 + n}$
*(3) Baumé 10° R = 12° 5 C.	$S = \frac{145.88}{145.88 - n}$	$S = \frac{145.88}{145.88 + (n - 10)}$
*(4) Baumé 14° R = 17° 5 C.	$S = \frac{146.78}{146.78 - n}$	$S = \frac{146.78}{146.78 + (n - 10)}$
*(5) Brix ; Official Prussian Hydrometer 12° 5 R = 15° 62 C.	$S = \frac{400}{400 - n}$	$S = \frac{400}{400 + n}$
*(6) Beck 10° R = 12° 5 C.	$S = \frac{170}{170 - n}$	$S = \frac{170}{170 + n}$

* According to Gerlach ; Dingler's Polytechnisches Journal, vol. 176, p. 444 (1865) ; vol. 181, p. 358 (1866) ; vol. 198 p. 313 (1870).

§ A very inconvenient temperature. We do not know whether it is being adhered to by Instrument Makers.

By means of first formula under (4). Gerlach calculated the following Table :*

SPECIFIC GRAVITIES CORRESPONDING TO n DEGREES BAUMÉ AT
 $14^{\circ}.0$ R = $17^{\circ}.5$ C.

n	Spec. Grav.	n	Spec. Grav.	n	Spec. Grav.
0	1.000	25	1.205	50	1.517
1	1.007	26	1.215	51	1.532
2	1.014	27	1.225	52	1.549
3	1.021	28	1.236	53	1.565
4	1.028	29	1.246	54	1.582
5	1.035	30	1.257	55	1.599
6	1.043	31	1.268	56	1.617
7	1.050	32	1.279	57	1.635
8	1.058	33	1.290	58	1.653
9	1.065	34	1.301	59	1.672
10	1.073	35	1.313	60	1.691
11	1.081	36	1.325	61	1.711
12	1.089	37	1.337	62	1.731
13	1.097	38	1.349	63	1.752
14	1.105	39	1.362	64	1.773
15	1.114	40	1.375	65	1.795
16	1.122	41	1.388	66	1.817
17	1.131	42	1.401	67	1.840
18	1.140	43	1.414	68	1.863
19	1.149	44	1.428	69	1.887
20	1.158	45	1.442	70	1.912
21	1.167	46	1.456	71	1.937
22	1.176	47	1.471	72	1.963
23	1.186	48	1.486	73	1.989
24	1.195	49	1.501	74	2.017

* Dingler's Polytechnisches Journal, vol. 198, p. 316 (1870). The original gives the specific gravity to 4 decimals.

RELATIONS IN A NUMBER OF AQUEOUS SOLUTIONS BETWEEN SPECIFIC GRAVITY AND PERCENTAGE OF SOLUTUM.

I. SULPHURIC ACID.

Tables "A" and "B" were deduced (by the Author) from Bineau's DIRECT DETERMINATIONS by a combination of algebraic and graphical interpolation. These tables go up to Spec. Grav. 1.7 only. The subsequent tables for stronger acids are transcriptions from the Originals quoted.

A. PERCENTAGE P OF H_2SO_4 IN FUNCTION OF TRUE* SPECIFIC GRAVITY ρ_{S_0} AT 0° , TAKEN IN REFERENCE TO WATER OF 0° AS A STANDARD.

* "True" means : reduced to the Vacuum.

ρ_{S_0}	0	1	2	3	4	5	6	7	8	9
1.00	0.00	0.14	0.28	0.42	0.56	0.70	0.83	0.97	1.11	1.25
1.01	1.39	1.53	1.67	1.81	1.94	2.08	2.22	2.36	2.50	2.63
1.02	2.78	2.91	3.05	3.19	3.32	3.46	3.60	3.74	3.87	4.01
1.03	4.15	4.29	4.42	4.56	4.69	4.83	4.97	5.10	5.24	5.37
1.04	5.52	5.65	5.79	5.92	6.06	6.20	6.33	6.47	6.60	6.74
1.05	6.87	7.01	7.14	7.28	7.41	7.55	7.68	7.82	7.95	8.09
1.06	8.22	8.36	8.49	8.62	8.76	8.89	9.03	9.16	9.29	9.43
1.07	9.56	9.69	9.83	9.96	10.09	10.23	10.36	10.49	10.62	10.76
1.08	10.89	11.02	11.16	11.29	11.42	11.55	11.68	11.82	11.95	12.08
1.09	12.21	12.34	12.47	12.61	12.74	12.87	13.00	13.13	13.26	13.39
1.10	13.53	13.66	13.79	13.92	14.05	14.18	14.31	14.44	14.57	14.70
1.11	14.83	14.96	15.09	15.22	15.35	15.48	15.61	15.74	15.87	16.00
1.12	16.12	16.25	16.38	16.51	16.64	16.77	16.90	17.03	17.16	17.29
1.13	17.41	17.54	17.67	17.79	17.92	18.05	18.18	18.31	18.43	18.56
1.14	18.69	18.81	18.94	19.07	19.19	19.32	19.45	19.58	19.70	19.83
1.15	19.96	20.08	20.21	20.33	20.46	20.59	20.71	20.84	20.96	21.09
1.16	21.22	21.34	21.47	21.59	21.72	21.84	21.97	22.09	22.22	22.34
1.17	22.47	22.59	22.72	22.84	22.96	23.09	23.21	23.34	23.46	23.58
1.18	23.71	23.83	23.95	24.08	24.20	24.32	24.45	24.57	24.69	24.81
1.19	24.94	25.06	25.19	25.31	25.43	25.55	25.67	25.80	25.92	26.04
1.20	26.16	26.28	26.41	26.53	26.65	26.78	26.90	27.02	27.14	27.27
1.21	27.39	27.51	27.63	27.76	27.88	28.00	28.12	28.24	28.37	28.49
1.22	28.61	28.73	28.85	28.97	29.09	29.22	29.34	29.46	29.58	29.70
1.23	29.82	29.94	30.06	30.18	30.30	30.43	30.55	30.67	30.79	30.91
1.24	31.03	31.15	31.27	31.40	31.52	31.64	31.76	31.88	32.01	32.13
1.25	32.25	32.37	32.49	32.60	32.72	32.84	32.96	33.08	33.19	33.31
1.26	33.43	33.55	33.67	33.79	33.91	34.03	34.14	34.26	34.38	34.50
1.27	34.62	34.74	34.85	34.97	35.09	35.21	35.32	35.44	35.56	35.67
1.28	35.79	35.91	36.02	36.14	36.25	36.37	36.49	36.60	36.72	36.83
1.29	36.95	37.07	37.18	37.30	37.42	37.54	37.65	37.77	37.89	38.00
1.30	38.12	38.23	38.35	38.46	38.57	38.69	38.80	38.91	39.02	39.14
1.31	39.25	39.36	39.47	39.59	39.70	39.81	39.92	40.03	40.15	40.26
1.32	40.37	40.48	40.60	40.71	40.82	40.94	41.05	41.16	41.27	41.39
1.33	41.50	41.61	41.72	41.83	41.94	42.06	42.17	42.28	42.39	42.50
1.34	42.61	42.72	42.83	42.94	43.05	43.17	43.28	43.39	43.50	43.61
ρ_{S_0}	0	1	2	3	4	5	6	7	8	9

$0^{\circ}S_0$	0	1	2	3	4	5	6	7	8	9
1.35	43.72	43.83	43.94	44.04	44.15	44.26	44.37	44.48	44.58	44.69
1.36	44.80	44.91	45.01	45.12	45.23	45.34	45.44	45.55	45.66	45.76
1.37	45.87	45.98	46.08	46.19	46.29	46.40	46.51	46.61	46.72	46.82
1.38	46.93	47.04	47.14	47.25	47.35	47.46	47.56	47.67	47.77	47.88
1.39	47.98	48.08	48.18	48.29	48.39	48.49	48.59	48.69	48.80	48.90
1.40	49.00	49.10	49.21	49.31	49.41	49.52	49.62	49.72	49.82	49.93
1.41	50.03	50.13	50.23	50.33	50.43	50.54	50.64	50.74	50.84	50.94
1.42	51.04	51.14	51.24	51.34	51.44	51.54	51.64	51.74	51.84	51.94
1.43	52.04	52.14	52.24	52.33	52.43	52.53	52.63	52.73	52.82	52.92
1.44	53.02	53.12	53.22	53.31	53.41	53.51	53.61	53.71	53.80	53.90
1.45	54.00	54.10	54.19	54.29	54.38	54.48	54.58	54.67	54.77	54.86
1.46	54.96	55.06	55.15	55.25	55.34	55.44	55.53	55.63	55.72	55.82
1.47	55.91	56.00	56.10	56.19	56.29	56.38	56.47	56.57	56.66	56.76
1.48	56.85	56.95	57.04	57.14	57.23	57.33	57.43	57.52	57.62	57.71
1.49	57.81	57.90	58.00	58.09	58.19	58.28	58.37	58.47	58.56	58.66
1.50	58.75	58.84	58.93	59.02	59.11	59.20	59.29	59.38	59.47	59.56
1.51	59.65	59.74	59.83	59.92	60.01	60.10	60.19	60.28	60.37	60.46
1.52	60.55	60.64	60.73	60.82	60.91	61.00	61.08	61.17	61.26	61.35
1.53	61.44	61.53	61.62	61.71	61.80	61.89	61.97	62.06	62.15	62.24
1.54	62.33	62.42	62.51	62.60	62.69	62.78	62.86	62.95	63.04	63.13
1.55	63.22	63.31	63.40	63.48	63.57	63.66	63.75	63.84	63.92	64.01
1.56	64.10	64.19	64.27	64.36	64.45	64.54	64.62	64.71	64.80	64.88
1.57	64.97	65.06	65.14	65.23	65.31	65.40	65.49	65.57	65.66	65.74
1.58	65.83	65.92	66.01	66.09	66.18	66.27	66.36	66.45	66.53	66.62
1.59	66.71	66.80	66.88	66.97	67.05	67.14	67.23	67.31	67.40	67.48
1.60	67.57	67.66	67.74	67.83	67.92	68.00	68.09	68.18	68.27	68.35
1.61	68.44	68.53	68.61	68.70	68.78	68.87	68.95	69.04	69.12	69.21
1.62	69.29	69.37	69.46	69.54	69.63	69.71	69.79	69.88	69.96	70.05
1.63	70.13	70.22	70.30	70.39	70.47	70.56	70.64	70.73	70.81	70.90
1.64	70.98	71.06	71.14	71.23	71.31	71.39	71.47	71.55	71.64	71.72
1.65	71.80	71.88	71.97	72.05	72.14	72.22	72.30	72.39	72.47	72.56
1.66	72.64	72.72	72.81	72.89	72.98	73.06	73.14	73.23	73.31	73.40
1.67	73.48	73.57	73.65	73.74	73.82	73.91	73.99	74.08	74.16	74.25
1.68	74.33	74.41	74.50	74.58	74.66	74.75	74.83	74.91	74.99	75.08
1.69	75.16	75.24	75.33	75.41	75.50	75.58	75.66	75.75	75.83	75.92
1.70	76.00	76.08	76.16	76.24	76.32	76.41	76.49	76.57	76.65	76.73
1.71	76.81	76.89	76.97	77.05	77.13	77.22	77.30	77.38	77.46	77.54
1.72	77.62	77.70	77.78	77.85	77.93	78.01	78.09	78.17	78.24	78.32
$0^{\circ}S_0$	0	1	2	3	4	5	6	7	8	9

$$H_2SO_4 \times 0.8164 = SO_3. \quad \text{Log.} = .9119 - 1.$$

$$SO_3 \times 1.2249 = H_2SO_4. \quad \text{Log.} = .0881.$$

AUXILIARY TABLES.

Table (1). For the reduction of a specific gravity "S" to the Vacuum; the weighings being supposed to be made at 15° and 760 mm. pressure. The reduced value (S) is less, than the value S found, by the quantity given in the table as Δ'

$$(S) = S - \Delta'; \quad S = (S) + \Delta'$$

S	Δ'	S	Δ'
1.00	Nil.	1.49	.000 6
1.08	.000 1	1.57	.000 7
1.16	.000 2	1.65	.000 8
1.24	.000 3	1.73	.000 9
1.33	.000 4	1.82	.001 0
1.41	.000 5	1.90	.001 1

For the given S substitute the nearest tabular S and adopt the correction given opposite to it. The result is right within $\pm .000\ 05$.

Table (2). To pass from $_{15}S$ to $_0S$, *i.e.*, from water of 15° to water of 0° as a standard. $_0S = _{15}S - \Delta''$ and $_{15}S = _0S + \Delta''$

S	Δ''	S	Δ''
(1.00	.000 711)		
0.98	.000 7	1.55	.001 1
1.13	.000 8	1.69	.001 2
1.27	.000 9	1.83	.001 3
1.41	.001 0	1.97	.001 4

Table (3). *DEPENDENCE OF THE SPECIFIC GRAVITY OF AN ACID ON ITS OWN TEMPERATURE.

To pass from ${}_cS_0$ to ${}_cS_t$ subtract Δ''' *per degree of temperature*. (Up to $t = 15^\circ$) Δ''' is a function of S , but independent of the temperature "c" of the water which serves as a standard.

Example: ${}_0S_0 - {}_0S_t = {}_{15}S_0 - {}_{15}S_t$

The Table gives the values of $1000 \times \Delta'''$, *i.e.*, the changes in S if the sp. gr. of water be taken as = 1000.

S_0	Second Decimal in S .									
	0	1	2	3	4	5	6	7	8	9
1.0	.05	.09	.13	.16	.20	.23	.27	.30	.33	.35
1.1	.38	.40	.44	.46	.48	.50	.53	.54	.56	.57
1.2	.59	.60	.61	.63	.64	.65	.66	.67	.67	.68
1.3	.70	.71	.73	.73	.74	.75	.75	.76	.77	.78
1.4	.79	.80	.80	.81	.81	.82	.83	.84	.85	.85
1.5	.86	.86	.86	.87	.87	.87	.87	.88	.88	.88
1.6	.88	.89	.89	.89	.90	.91	.91	.91	.91	.91
1.7	.91									

Ex.—For $S = 1.34$, the correction is $0.001 \times 0.74 = 0.00074$ for 1° and consequently $5 \times 0.00074 = 0.0037$ for 5° .

EXAMPLES TO SHOW HOW THE ABOVE TABLES ARE USED.

I. 1 vol. of water of 15° weighs 1.0000. The same volume of a sulphuric acid of 10° weighs 1.5000 in air of 15° and 760mm. To find the percentage of H_2SO_4 , apply the following corrections:—

- | | |
|--|----------|
| (1) To reduce to the vacuum, add (by Table 1), | — .000 6 |
| (2) To pass from ${}_{15}S_{10}$ to ${}_0S_{10}$, add (by Table 2), | — .001 1 |
| (3) To pass from ${}_0S_{10}$ to ${}_0S_0$, add (by Table 3), | + .008 6 |

Sum of corrections,	+ .006 9
---------------------	----------

Hence ${}_0S_0 = 1.5069$; and hence, by Table A, $p = 59.37$; or rather 59.4 because the second place is utterly uncertain.

II. The *un*-corrected value for ${}_{15}S_{15}$ is 1.5. Find p . In this case we have

- | | |
|--|----------|
| (1) for the vacuum-correction, | — .000 6 |
| (2) To pass from ${}_{15}S_{15}$ to ${}_0S_{15}$, | — .001 1 |
| (3) To pass from ${}_0S_{15}$ to ${}_0S_0$, | + .012 9 |

Sum of corrections,	+ .011 2
---------------------	----------

Hence ${}_0S_0 = 1.5112$; and thence, by Table A, $p = 59.76$; or rather 59.8.

For cases like the second, the following Table B gives p directly or by an easy interpolation:—

* Derived from a few data of Bineau's by graphic interpolation. His numbers only to the third place.

B. RELATION BETWEEN $^{15}\text{S}_{15}$ (NOT REDUCED TO THE VACUUM) AND
PERCENTAGE p OF H_2SO_4 .

$^{15}\text{S}_{15}$	p	Diff.	ΔS for $\Delta p=1\%$	$^{15}\text{S}_{15}$	p	Diff.	ΔS for $\Delta p=1\%$
1.00	0.00			1.35	44.78	1.11	.0090
1.01	1.46	1.46	.0068	1.36	45.86	1.08	.0093
1.02	2.92	1.46	.0068	1.37	46.93	1.07	.0093
1.03	4.35	1.43	.0070	1.38	47.99	1.06	.0094
1.04	5.80	1.45	.0069	1.39	49.04	1.05	.0095
1.05	7.21	1.41	.0071	1.40	50.06	1.02	.0098
1.06	8.62	1.41	.0071	1.41	51.09	1.03	.0097
1.07	10.02	1.40	.0071	1.42	52.09	1.00	.0100
1.08	11.41	1.39	.0072	1.43	53.09	1.00	.0100
1.09	12.77	1.36	.0074	1.44	54.07	0.98	.0102
1.10	14.14	1.37	.0073	1.45	55.05	0.98	.0102
1.11	15.49	1.35	.0074	1.46	56.00	0.95	.0105
1.12	16.82	1.33	.0075	1.47	56.95	0.95	.0105
1.13	18.16	1.34	.0075	1.48	57.88	0.93	.0108
1.14	19.47	1.31	.0076	1.49	58.83	0.95	.0105
1.15	20.77	1.30	.0077	1.50	59.76	0.93	.0108
1.16	22.06	1.29	.0078	1.51	60.66	0.90	.0111
1.17	23.34	1.28	.0078	1.52	61.55	0.89	.0112
1.18	24.60	1.26	.0079	1.53	62.44	0.89	.0112
1.19	25.86	1.26	.0079	1.54	63.33	0.89	.0112
1.20	27.10	1.24	.0081	1.55	64.22	0.89	.0112
1.21	28.34	1.24	.0081	1.56	65.09	0.87	.0115
1.22	29.58	1.24	.0081	1.57	65.96	0.87	.0115
1.23	30.81	1.23	.0081	1.58	66.81	0.85	.0118
1.24	32.03	1.22	.0082	1.59	67.69	0.88	.0114
1.25	33.27	1.24	.0081	1.60	68.55	0.86	.0116
1.26	34.45	1.18	.0085	1.61	69.42	0.87	.0115
1.27	35.65	1.20	.0083	1.62	70.26	0.84	.0119
1.28	36.83	1.18	.0085	1.63	71.10	0.84	.0119
1.29	37.99	1.16	.0086	1.64	71.95	0.85	.0118
1.30	39.17	1.18	.0085	1.65	72.76	0.81	.0123
1.31	40.30	1.13	.0088	1.66	73.60	0.84	.0119
1.32	41.42	1.12	.0089	1.67	74.44	0.84	.0119
1.33	42.56	1.14	.0088	1.68	75.29	0.85	.0118
1.34	43.67	1.11	.0090	1.69	76.12	0.83	.0120
				1.70	76.96	0.84	.0119

Examples.—To $S=1.546$ corresponds the percentage
 $63.33 + 0.6 \times 0.89 = 63.33 + 0.53 = 63.86$; or rather 63.9.

To the percentage 60.0 (as the S for 59.76 is 1.5) corresponds
 $S = 1.50 + .24 \times .0111 = 1.50 + .0027 = 1.5027$; or rather 1.503.

C. RELATION IN SULPHURIC ACID BETWEEN SPECIFIC GRAVITY S OF degrees Baumé (°B), AND PERCENTAGE OF H_2SO_4 ; according to Bineau (Ann. de Chim. et de Phys. (3) vol. 26, page 125; Year 1849.)

°B	Spec. Grav.	Diff.	Percents. H_2SO_4 if Temp. of Acid = 0°	Diff.	Percents. H_2SO_4 if Temp. of Acid = 15°	Diff.
59	1.691		75.2		76.3	
60	1.711	.020	76.9	1.7	78.0	1.7
61	1.732	.021	78.6	1.7	79.8	1.8
62	1.753	.021	80.4	1.8	81.7	1.9
63	1.774	.021	82.4	2.0	83.9	2.2
64	1.796	.022	84.6	2.2	86.3	2.4
65.0	1.819	.023	87.4	2.8	89.5	3.2
65.5	1.830	.011	89.1	1.7	91.8	2.3
65.8	1.837	.007	90.4	1.3	94.5	2.7
66.0	1.842	.005	91.3	0.9	100.0	5.5
66.2	1.846	.004	92.5	1.2		
66.4	1.852	.006	95.0	2.5		
66.6	1.857	.005	100.0	5.0		

D. SPECIFIC GRAVITIES OF MOST HIGHLY CONCENTRATED SULPHURIC ACIDS, according to G. Lunge and Naef. Temperature of Acid = 15°, that of Water = 4°.

a. Direct Observations.

Percents. of H_2SO_4	$4S_{15}$
90.20	1.8195
91.48	1.8271
92.83	1.8334
94.84	1.8387
95.97	1.8406
97.70	1.8413
98.39	1.8406
98.66	1.8409
99.47	1.8395
100.00	1.8384

b. Working Table, by Interpolation from a.

Percents. of H_2SO_4	Spec. Grav. $4S_{15}$	Diff.
90	1.8185	—
91	1.8241	.0056
92	1.8294	.0053
93	1.8339	.0045
94	1.8372	.0033
95	1.8390	.0018
96	1.8406	.0016
97	1.8410	.0004
98	1.8412	.0002
99	1.8403	-.0009
100	1.8384	-.0019

The above Table of Lunge's and the following Tables on Hydrochloric Acid and Nitric Acid are extracted from Lunge and Hurter's "Alkali Makers' Pocket-Book":—

II. HYDROCHLORIC ACID.

(a) *Calculated after Kolb's determinations.*

$15S_{15}$	Percents. of HCl.	$15S_{15}$	Percents. of HCl.
1.00	0.00	1.10	20.06
1.01	2.12	1.11	22.06
1.02	4.11	1.12	24.05
1.03	6.11	1.13	26.04
1.04	8.10	1.14	28.04
1.05	10.09	1.15	30.03
1.06	12.09	1.16	32.02
1.07	14.08	1.17	34.02
1.08	16.07	1.18	36.01
1.09	18.07	1.19	38.01
1.10	20.06	1.20	40.00

Approximately :

$\Delta p = 200 \times \Delta S$, and
 $\Delta S = 0.005 \times \Delta p$; or
 percentages of HCl are
 equal to the number of
 degrees Twaddle.

The following Table, which is based on determinations made in my laboratory by Mr. Kling, covers only a small interval, but, as far as it goes, probably affords a higher degree of precision than Kolb's:—

TABLE FOR FINDING THE PERCENTAGE p OF HCl FROM THE SPECIFIC GRAVITY $15S$ AT 10° TO 20° .

(b) *Calculated by W. Dittmar, after determinations by Arch. Kling.*

$15S_{15}$	p	Δp	For $\Delta t = 1^\circ$, $\Delta S =$ (0.0001 $\times$)	$15S_{15}$	p	Δp	For $\Delta t = 1^\circ$, $\Delta S =$ (0.0001 $\times$)
1.085	16.96		3.0	1.100	19.85	.19	3.5
1.086	17.16	.20	3.0	1.101	20.04	.19	3.6
1.087	17.35	.19	3.0	1.102	20.24	.20	3.6
1.088	17.54	.19	3.1	1.103	20.43	.19	3.6
1.089	17.73	.19	3.1	1.104	20.62	.19	3.7
1.090	17.93	.20	3.2	1.105	20.81	.19	3.7
1.091	18.12	.19	3.2	1.106	21.01	.20	3.8
1.092	18.31	.19	3.2	1.107	21.20	.19	3.8
1.093	18.50	.19	3.3	1.108	21.39	.19	3.8
1.094	18.70	.20	3.3	1.109	21.58	.19	3.9
1.095	18.89	.19	3.3	1.110	21.78	.20	3.9
1.096	19.08	.19	3.4	1.111	21.97	.19	3.9
1.097	19.27	.19	3.4	1.112	22.16	.19	4.0
1.098	19.47	.20	3.5	1.113	22.35	.19	4.0
1.099	19.66	.19	3.5	1.114	22.55	.20	4.1

Water of 15° is the standard for S throughout; the values S are *not* reduced to the Vacuum.

III. HYDROBROMIC ACID.

According to Topsöe; Ber. der
Deutschen Chem. Gesellschaft,
1870, 404.

t	t _S	% of HBr.
14°	1.055	7.67
14°	1.075	10.19
14°	1.089	11.94
14°	1.097	12.96
14°	1.118	15.37
14°	1.131	16.92
14°	1.164	20.65
13°	1.200	24.35
13°	1.232	27.62
13°	1.253	29.68
13°	1.302	33.84
13°	1.335	36.67
13°	1.349	37.86
13°	1.368	39.13
13°	1.419	43.12
13°	1.431	43.99
13°	1.438	44.62
14°	1.451	45.45
13°	1.460	46.09
14°	1.485	47.87
14°	1.490	48.17

IV. HYDRIODIC ACID.

According to Topsöe; Ber. der
Deutschen Chem. Gesellschaft,
1870, 403.

t	t _S	% of HI.
13.5°	1.017	2.29
13.5°	1.052	7.02
13.5°	1.077	10.15
13°	1.095	12.21
13.5°	1.102	13.09
13.5°	1.126	15.73
13.5°	1.164	19.97
13.8°	1.191	22.63
13.8°	1.225	25.86
13.5°	1.254	28.41
13.5°	1.274	30.20
13°	1.309	33.07
13°	1.347	36.07
13°	1.382	38.68
13°	1.413	40.45
13°	1.451	43.39
13°	1.487	45.71
13°	1.528	48.22
13.5°	1.542	49.13
13°	1.573	50.75
12.5°	1.603	52.43
14°	1.630	53.93
13.7°	1.674	56.15
13°	1.696	57.28
12.5°	1.703	57.42
13.7°	1.706	57.64
12°	1.708	57.74

V. NITRIC ACID.

After determinations by Kolb.

$15S_{15}$	Percents p of HNO_3	Diff.	ΔS for $\Delta p = 1\%$	$15S_{15}$	p	Diff.	ΔS for $\Delta p = 1\%$
1.00	0.00			1.30	47.57	1.53	.0065
1.01	1.75	1.75	.0057	1.31	49.10	1.53	.0065
1.02	3.50	1.75	.0057	1.32	50.63	1.53	.0065
1.03	5.25	1.75	.0057	1.33	52.24	1.61	.0062
1.04	6.95	1.70	.0059	1.34	53.94	1.70	.0059
1.05	8.59	1.64	.0061	1.35	55.64	1.70	.0059
1.06	10.23	1.64	.0061	1.36	57.42	1.78	.0056
1.07	11.88	1.65	.0061	1.37	59.21	1.79	.0056
1.08	13.52	1.64	.0061	1.38	61.00	1.79	.0056
1.09	15.16	1.64	.0061	1.39	62.95	1.95	.0051
1.10	16.80	1.64	.0061	1.40	64.90	1.95	.0051
1.11	18.35	1.55	.0065	1.41	67.12	2.22	.0045
1.12	19.89	1.54	.0065	1.42	69.34	2.22	.0045
1.13	21.44	1.55	.0065	1.43	71.83	2.49	.0040
1.14	22.98	1.54	.0065	1.44	74.59	2.76	.0036
1.15	24.53	1.55	.0065	1.45	77.36	2.77	.0036
1.16	26.08	1.55	.0065	1.46	80.13	2.77	.0036
1.17	27.62	1.54	.0065	1.47	82.90	2.77	.0036
1.18	29.17	1.55	.0065	1.48	85.66	2.76	.0036
1.19	30.71	1.54	.0065	1.49	88.43	2.77	.0036
1.20	32.26	1.55	.0065	1.50	91.20	2.77	.0036
1.21	33.79	1.53	.0065	1.51	94.13	2.93	.0034
1.22	35.32	1.53	.0065	1.52	97.06	2.93	.0034
1.23	36.85	1.53	.0065	1.53	100.00	2.94	.0034
1.24	38.38	1.53	.0065				
1.25	39.91	1.53	.0065				
1.26	41.44	1.53	.0065				
1.27	42.97	1.53	.0065				
1.28	44.50	1.53	.0065				
1.29	46.04	1.54	.0065				

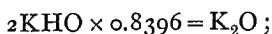
$$2HNO_3 \times 0.8572 = N_2O_5; \text{Log.} = .9331 - 1.$$

$$N_2O_5 \times 1.1666 = 2HNO_3; \text{Log.} = 1.0669.$$

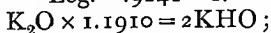
VI. SPECIFIC GRAVITIES OF CAUSTIC POTASH LEYS, AT 15°.

(Calculated by Gerlach from Determinations by Tünnermann and by H. Schiff.)

Per- cents of KHO	Spec. Gr.	Per- cents of KHO	Spec. Gr.
0	1.000	35	1.349
1	1.009	36	1.361
2	1.017	37	1.374
3	1.025	38	1.387
4	1.033	39	1.400
5	1.041	40	1.412
6	1.049	41	1.425
7	1.058	42	1.438
8	1.065	43	1.450
9	1.074	44	1.462
10	1.083	45	1.475
11	1.092	46	1.488
12	1.101	47	1.499
13	1.110	48	1.511
14	1.119	49	1.525
15	1.128	50	1.539
16	1.137	51	1.552
17	1.146	52	1.565
18	1.155	53	1.578
19	1.166	54	1.590
20	1.177	55	1.604
21	1.188	56	1.618
22	1.198	57	1.630
23	1.209	58	1.642
24	1.220	59	1.655
25	1.230	60	1.667
26	1.241	61	1.681
27	1.252	62	1.695
28	1.264	63	1.705
29	1.276	64	1.718
30	1.288	65	1.729
31	1.300	66	1.740
32	1.311	67	1.754
33	1.324	68	1.768
34	1.336	69	1.780
		70	1.790



$$\text{Log.} = .9241 - 1.$$

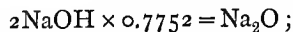


$$\text{Log.} = 0.750.$$

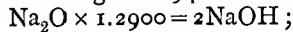
VII. SPECIFIC GRAVITIES OF CAUSTIC SODA LEYS, AT 15°.

(Calculated by Gerlach from Determinations by H. Schiff.)

Per- cents of NaOH	Spec. Gr.	Per- cents of NaOH	Spec. Gr.
0	1.000	35	1.384
1	1.012	36	1.395
2	1.023	37	1.405
3	1.035	38	1.415
4	1.046	39	1.426
5	1.058	40	1.437
6	1.070	41	1.447
7	1.081	42	1.457
8	1.092	43	1.468
9	1.103	44	1.478
10	1.115	45	1.488
11	1.126	46	1.499
12	1.137	47	1.509
13	1.148	48	1.519
14	1.159	49	1.529
15	1.170	50	1.540
16	1.181	51	1.550
17	1.192	52	1.560
18	1.202	53	1.570
19	1.213	54	1.580
20	1.225	55	1.591
21	1.236	56	1.601
22	1.247	57	1.611
23	1.258	58	1.622
24	1.269	59	1.633
25	1.279	60	1.643
26	1.290	61	1.654
27	1.300	62	1.664
28	1.310	63	1.674
29	1.321	64	1.684
30	1.332	65	1.695
31	1.343	66	1.705
32	1.353	67	1.715
33	1.363	68	1.726
34	1.374	69	1.737
		70	1.748



$$\text{Log.} = .8894 - 1.$$



$$\text{Log.} = 1.1106.$$

VIII. SPECIFIC GRAVITIES OF AQUEOUS AMMONIAS AT 15°, WATER OF 15° = 1. According to Lunge and Wiernik.*

$_{15}S_{16}$	Per- cents of NH_3	One Litre of 15° contains Grammes of NH_3	Correction of $_{15}S$ for $\pm 1^\circ$ in the temp of the Aqueous Ammonia. **	$_{15}S_{16}$	Per- cents of NH_3	One Litre of 15° contains Grammes of NH_3	Correction of $_{15}S$ for $\pm 1^\circ$ in the temp of the Aqueous Ammonia. **
1.000	0.00	0.0	0.00018	0.940	15.63	146.9	0.00039
0.998	0.45	4.5	0.00018	0.938	16.22	152.1	0.00040
0.996	0.91	9.1	0.00019	0.936	16.82	157.4	0.00041
0.994	1.37	13.6	0.00019	0.934	17.42	162.7	0.00041
0.992	1.84	18.2	0.00020	0.932	18.03	168.1	0.00042
0.990	2.31	22.9	0.00020	0.930	18.64	173.4	0.00042
0.988	2.80	27.7	0.00021	0.928	19.25	178.6	0.00043
0.986	3.30	32.5	0.00021	0.926	19.87	184.2	0.00044
0.984	3.80	37.4	0.00022	0.924	20.49	189.3	0.00045
0.982	4.30	42.2	0.00022	0.922	21.12	194.7	0.00046
0.980	4.80	47.0	0.00023	0.920	21.75	200.1	0.00047
0.978	5.30	51.8	0.00023	0.918	22.39	205.6	0.00048
0.976	5.80	56.6	0.00024	0.916	23.03	210.9	0.00049
0.974	6.30	61.4	0.00024	0.914	23.68	216.3	0.00050
0.972	6.80	66.1	0.00025	0.912	24.33	221.9	0.00051
0.970	7.31	70.9	0.00025	0.910	24.99	227.4	0.00052
0.968	7.82	75.7	0.00026	0.908	25.65	232.9	0.00053
0.966	8.33	80.5	0.00026	0.906	26.31	238.3	0.00054
0.964	8.84	85.2	0.00027	0.904	26.98	243.9	0.00055
0.962	9.35	89.9	0.00028	0.902	27.65	249.4	0.00056
0.960	9.91	95.1	0.00029	0.900	28.33	255.0	0.00057
0.958	10.47	100.3	0.00030	0.898	29.01	260.5	0.00058
0.956	11.03	105.4	0.00031	0.896	29.69	266.0	0.00059
0.954	11.60	110.7	0.00032	0.894	30.37	271.5	0.00060
0.952	12.17	115.9	0.00033	0.892	31.05	277.0	0.00060
0.950	12.74	121.0	9.00034	0.890	31.75	282.6	0.00061
0.948	13.31	126.2	0.00035	0.888	32.50	288.6	0.00062
0.946	13.88	131.3	0.00036	0.886	33.25	294.6	0.00063
0.944	14.46	136.5	0.00037	0.884	34.10	301.4	0.00064
0.942	15.04	141.7	0.00038	0.882	34.95	308.3	0.00065

* Zeitschrift für Angewandte Chemie, 1889; Heft 7.—Kindly communicated by Professor Lunge.

** For the interval 13° to 15°.

IX. TABLE GIVING THE RELATION IN AQUEOUS ETHYL-ALCOHOL BETWEEN SPECIFIC GRAVITY AT 15°C. ON THE ONE HAND, AND THE PERCENTAGE BY WEIGHT OF REAL ALCOHOL, OR THE SPECIFIC GRAVITY AT ANY TEMPERATURE FROM 10° TO 20° ON THE OTHER.

Calculated after Mendelejeff's Determinations, by W. Dittmar.

Water of 15°C. = 1000.

Spec. Grav. at 15. ^o 15S ₁₅	Percent- age p.	Δp for $\Delta S =$ 1	Diff. in Spec. Grav. for		Spec. Grav. at 15. ^o 15S ₁₅	Percent- age p.	Δp for $\Delta S =$ 1	Diff. in Spec. Grav. for	
			$\Delta p =$ 1	$\Delta t = 1^{\circ}$				$\Delta p =$ 1	$\Delta t = 1^{\circ}$
1000	0.00			0.144	975	17.25	.84	1.19	.330
999	0.53	.53	1.89	.146	974	18.08	.83	1.20	.348
998	1.06	.53	1.89	.147	973	18.91	.83	1.20	.367
997	1.61	.55	1.82	.150	972	19.73	.82	1.22	.387
996	2.16	.55	1.82	.152	971	20.54	.81	1.23	.405
995	2.73	.57	1.75	.154	970	21.35	.81	1.23	.421
994	3.30	.57	1.75	.157	969	22.13	.78	1.28	.434
993	3.91	.61	1.64	.161	968	22.89	.76	1.32	.450
992	4.51	.60	1.67	.166	967	23.65	.76	1.32	.466
991	5.12	.61	1.64	.170	966	24.39	.74	1.35	.480
990	5.77	.65	1.54	.174	965	25.13	.74	1.35	.496
989	6.43	.66	1.52	.180	964	25.84	.71	1.41	.510
988	7.09	.66	1.52	.186	963	26.53	.69	1.45	.522
987	7.78	.69	1.45	.192	962	27.23	.70	1.43	.535
986	8.49	.71	1.41	.199	961	27.90	.67	1.49	.548
985	9.20	.71	1.41	.206	960	28.55	.65	1.54	.560
984	9.96	.76	1.32	.214	959	29.21	.66	1.52	.570
983	10.72	.76	1.32	.223	958	29.84	.63	1.59	.581
982	11.50	.78	1.28	.231	957	30.46	.62	1.61	.593
981	12.28	.78	1.28	.242	956	31.09	.63	1.59	.604
980	13.10	.82	1.22	.254	955	31.69	.60	1.67	.614
979	13.92	.82	1.22	.267	954	32.30	.61	1.64	.623
978	14.74	.82	1.22	.281	953	32.88	.58	1.72	.632
977	15.58	.84	1.19	.296	952	33.45	.57	1.75	.641
976	16.41	.83	1.20	.311	951	34.03	.58	1.72	.650

ETHYL-ALCOHOL.

Spec. Grav. at 15° 15S15	Percent- age P.	Δp for $\Delta S = 1$	Diff. in Spec. Grav. for		Spec. Grav. at 15° 15S15	Percent- age P.	Δp for $\Delta S = 1$	Diff. in Spec. Grav. for	
			$\Delta p = 1$	$\Delta t = 1^\circ$				$\Delta p = 1$	$\Delta t = 1^\circ$
950	34.59	.56	1.79	.657	915	51.68	.45	2.22	.800
949	35.15	.56	1.79	.666	914	52.13	.45	2.22	.801
948	35.70	.55	1.82	.673	913	52.58	.45	2.22	.802
947	36.25	.55	1.82	.680	912	53.02	.44	2.27	.803
946	36.78	.53	1.89	.687	911	53.47	.45	2.22	.804
945	37.31	.53	1.89	.694	910	53.91	.44	2.27	.806
644	37.84	.53	1.89	.700	909	54.35	.44	2.27	.807
943	38.36	.52	1.92	.707	908	54.80	.45	2.22	.808
942	38.88	.52	1.92	.712	907	55.24	.44	2.27	.809
941	39.39	.51	1.96	.718	906	55.68	.44	2.27	.810
940	39.90	.51	1.96	.723	905	56.12	.44	2.27	.811
939	40.40	.50	2.00	.727	904	56.56	.44	2.27	.812
938	40.90	.50	2.00	.733	903	57.00	.44	2.27	.813
937	41.40	.50	2.00	.737	902	57.43	.43	2.33	.813
936	41.89	.49	2.04	.741	901	57.87	.44	2.27	.813
935	42.38	.49	2.04	.746	900	58.31	.44	2.27	.814
934	42.86	.48	2.08	.750	899	58.74	.43	2.33	.815
933	43.35	.49	2.04	.753	898	59.18	.44	2.27	.816
932	43.83	.48	2.08	.756	897	59.61	.43	2.33	.817
931	44.30	.47	2.13	.760	896	60.05	.44	2.27	.817
930	44.78	.48	2.08	.763	895	60.48	.43	2.33	.817
929	45.25	.47	2.13	.767	894	60.91	.43	2.33	.818
928	45.73	.48	2.08	.770	893	61.35	.44	2.27	.819
927	46.20	.47	2.13	.773	892	61.78	.43	2.33	.819
926	46.66	.46	2.17	.774	891	62.21	.43	2.33	.820
925	47.13	.47	2.13	.778	890	62.64	.43	2.33	.820
924	47.59	.46	2.17	.781	889	63.07	.43	2.33	.820
923	48.05	.46	2.17	.783	888	63.50	.43	2.33	.820
922	48.51	.46	2.17	.786	887	63.93	.43	2.33	.821
921	48.96	.45	2.22	.788	886	64.36	.43	2.33	.822
920	49.42	.46	2.17	.790	885	64.79	.43	2.33	.822
919	49.88	.46	2.17	.792	884	65.21	.42	2.38	.823
918	50.33	.45	2.22	.794	883	65.64	.43	2.33	.824
917	50.78	.45	2.22	.796	882	66.07	.43	2.33	.825
916	51.23	.45	2.22	.798	881	66.49	.42	2.38	.826

Spec. Grav. at 15. ^o 15S15	Percent- age P.	Δp for $\Delta S =$ I	Diff. in Spec. Grav. for		Spec. Grav. at 15. ^o 15S15	Percent- age P.	Δp for $\Delta S =$ I	Diff. in Spec. Grav. for	
			$\Delta p =$ I	$\Delta t = 1^{\circ}$				$\Delta p =$ I	$\Delta t = 1^{\circ}$
880	66.92	.43	2.33	.825	845	81.46	.40	2.50	.854
879	67.34	.42	2.38	.826	844	81.86	.40	2.50	.855
878	67.77	.43	2.33	.826	843	82.26	.40	2.50	.856
877	68.20	.43	2.33	.827	842	82.66	.40	2.50	.857
876	68.62	.42	2.38	.828	841	83.06	.40	2.50	.858
875	69.04	.42	2.38	.829	840	83.46	.40	2.50	.859
874	69.46	.42	2.38	.830	839	83.86	.40	2.50	.860
873	69.88	.42	2.38	.830	838	84.26	.40	2.50	.860
872	70.30	.42	2.38	.830	837	84.65	.39	2.56	.860
871	70.73	.43	2.33	.831	836	85.05	.40	2.50	.861
870	71.15	.42	2.38	.832	835	85.44	.39	2.56	.862
869	71.57	.42	2.38	.833	834	85.83	.39	2.56	.862
868	71.99	.42	2.38	.834	833	86.22	.39	2.56	.862
867	72.40	.41	2.44	.835	832	86.61	.39	2.56	.863
866	72.82	.42	2.38	.836	831	87.00	.39	2.56	.863
865	73.24	.42	2.38	.836	830	87.38	.38	2.63	.864
864	73.66	.42	2.38	.837	829	87.77	.39	2.56	.865
863	74.08	.42	2.38	.838	828	88.15	.38	2.63	.865
862	74.49	.41	2.44	.839	827	88.53	.38	2.63	.865
861	74.90	.41	2.44	.840	826	88.91	.38	2.63	.865
860	75.32	.42	2.38	.840	825	89.29	.38	2.63	.865
859	75.73	.41	2.44	.840	824	89.67	.38	2.63	.866
858	76.15	.42	2.38	.841	823	90.05	.38	2.63	.866
857	76.56	.41	2.44	.842	822	90.42	.37	2.70	.866
856	76.97	.41	2.44	.842	821	90.79	.37	2.70	.866
855	77.38	.41	2.44	.844	820	91.16	.37	2.70	.865
854	77.79	.41	2.44	.845	819	91.53	.37	2.70	.864
853	78.21	.42	2.38	.846	818	91.89	.36	2.78	.864
852	78.62	.41	2.44	.847	817	92.26	.37	2.70	.863
851	79.02	.40	2.50	.848	816	92.62	.36	2.78	.862
850	79.43	.41	2.44	.848	815	92.98	.36	2.78	.862
849	79.84	.41	2.44	.849	814	93.34	.36	2.78	.861
848	80.24	.40	2.50	.850	813	93.70	.36	2.78	.861
847	80.65	.41	2.44	.852	812	94.05	.35	2.86	.860
846	81.06	.41	2.44	.853	811	94.41	.36	2.78	.859

Spec. Grav. at 15°. $_{15}S_{15}$	Percent-age p.	Δp for $\Delta S = 1$	Diff. in Spec. Grav. for		Spec. Grav. at 15°. $_{15}S_{15}$	Percent-age p.	Δp for $\Delta S = 1$	Diff. in Spec. Grav. for	
			$\Delta p = 1$	$\Delta t = 1^\circ$				$\Delta p = 1$	$\Delta t = 1^\circ$
810	94.76	.35	2.86	.859	800	98.16	.33	3.03	.850
809	95.11	.35	2.86	.859	799	98.49	.33	3.03	.849
808	95.45	.34	2.94	.858	798	98.81	.32	3.13	.848
807	95.80	.35	2.86	.857	797	99.14	.33	3.03	.847
806	96.15	.35	2.86	.856	796	99.46	.32	3.13	.846
805	96.48	.33	3.03	.856	795	99.78	.32	3.13	.845
804	96.82	.34	2.94	.855	794	100.10	.32	3.13	.844
803	97.16	.34	2.94	.854	794.32	100			
802	97.49	.33	3.03	.852					
801	97.83	.34	2.94	.851					

Referring to water of 4°C . as = 100 000, the specific gravity ${}_4S_t$ of absolute alcohol, according to Mendelejeff, is

$${}_4S_t = 80\ 625 - 83.4\ t - 0.029\ t^2;$$

hence (by Mendelejeff's computation) we have for the

Specific Gravities of Absolute Alcohol—

t	0°	5°	10°	15°	20°	25°	30°
${}_4S_t$	806 25	802 07	797 88	793 67	789 45	785 22	780 96

In regard to CHANGE OF STANDARD, see p. 97.

NOTE.—The specific gravities in our table, though given to only 3 decimals, are correct arithmetically, to within *less* than .000 02. Thus, for instance, the specific gravity of 87 per cent. Alcohol is .831 00 $\pm$.000 02 at the worst.

Table X.* ETHYL-ALCOHOL; TO PASS FROM PERCENTS OF ALCOHOL BY WEIGHT, p , TO PERCENTS BY VOLUME (p), AT 15°C.

Percents by		Diff.	Percents by		Diff.	Percents by		Diff.
Weight p	Volume (p)		Weight p	Volume (p)		Weight p	Volume (p)	
0	0.00		35	41.83	1.12	70	76.91	.89
1	1.26	1.26	36	42.94	1.11	71	77.80	.89
2	2.51	1.25	37	44.05	1.11	72	78.68	.88
3	3.76	1.25	38	45.15	1.10	73	79.55	.87
4	5.00	1.24	39	46.24	1.09	74	80.42	.87
5	6.24	1.24	40	47.33	1.09	75	81.27	.85
6	7.48	1.24	41	48.41	1.08	76	82.13	.86
7	8.71	1.23	42	49.48	1.07	77	82.97	.84
8	9.94	1.23	43	50.55	1.07	78	83.81	.84
9	11.16	1.22	44	51.61	1.06	79	84.64	.83
10	12.39	1.23	45	52.66	1.05	80	85.47	.83
11	13.61	1.22	46	53.71	1.05	81	86.28	.81
12	14.83	1.22	47	54.75	1.04	82	87.09	.81
13	16.04	1.21	48	55.78	1.03	83	87.90	.81
14	17.25	1.21	49	56.81	1.03	84	88.69	.79
15	18.46	1.21	50	57.83	1.02	85	89.47	.78
16	19.67	1.21	51	58.85	1.02	86	90.25	.78
17	20.87	1.20	52	59.85	1.00	87	91.02	.77
18	22.07	1.20	53	60.86	1.01	88	91.78	.76
19	23.27	1.20	54	61.85	.99	89	92.52	.74
20	24.47	1.20	55	62.84	.99	90	93.26	.74
21	25.66	1.19	56	63.82	.98	91	93.99	.73
22	26.84	1.18	57	64.80	.98	92	94.71	.72
23	28.02	1.18	58	65.77	.97	93	95.42	.71
24	29.20	1.18	59	66.73	.96	94	96.11	.69
25	30.38	1.18	60	67.69	.96	95	96.79	.68
26	31.55	1.17	61	68.64	.95	96	97.46	.67
27	32.71	1.16	62	69.59	.95	97	98.12	.66
28	33.87	1.16	63	70.52	.93	98	98.76	.64
29	35.02	1.15	64	71.46	.94	99	99.39	.63
30	36.17	1.15	65	72.38	.92	100	100.00	.61
31	37.32	1.15	66	73.30	.92			
32	38.45	1.13	67	74.21	.91			
33	39.58	1.13	68	75.12	.91			
34	40.71	1.13	69	76.02	.90			

* Calculated by W. Dittmar.

Table XI.* TO PASS FROM PERCENTS BY VOLUME AT 15°C., (p). TO
PERCENTS BY WEIGHT p.

Percents by		Diff.	Percents by		Diff.	Percents by		Diff.
Volume (p)	Weight p		Volume (p)	Weight p		Volume (p)	Weight p	
0	0.00		35	28.98	.87	70	62.44	1.06
1	0.80	.80	36	29.85	.87	71	63.51	1.07
2	1.59	.79	37	30.72	.87	72	64.59	1.08
3	2.39	.80	38	31.60	.88	73	65.68	1.09
4	3.20	.81	39	32.48	.88	74	66.77	1.09
5	4.00	.80	40	33.37	.89	75	67.87	1.10
6	4.81	.81	41	34.26	.89	76	68.98	1.11
7	5.62	.81	42	35.16	.90	77	70.10	1.12
8	6.43	.81	43	36.05	.89	78	71.23	1.13
9	7.24	.81	44	36.96	.91	79	72.37	1.14
10	8.05	.81	45	37.87	.91	80	73.52	1.15
11	8.87	.82	46	38.78	.91	81	74.68	1.16
12	9.68	.81	47	39.70	.92	82	75.85	1.17
13	10.50	.82	48	40.62	.92	83	77.03	1.18
14	11.32	.82	49	41.55	.93	84	78.23	1.20
15	12.14	.82	50	42.49	.94	85	79.43	1.20
16	12.97	.83	51	43.43	.94	86	80.65	1.22
17	13.79	.82	52	44.37	.94	87	81.88	1.23
18	14.62	.83	53	45.32	.95	88	83.13	1.25
19	15.45	.83	54	46.28	.96	89	84.40	1.27
20	16.27	.82	55	47.24	.96	90	85.68	1.28
21	17.11	.84	56	48.21	.97	91	86.98	1.30
22	17.94	.83	57	49.19	.98	92	88.30	1.32
23	18.77	.83	58	50.17	.98	93	89.64	1.34
24	19.61	.84	59	51.15	.98	94	91.01	1.37
25	20.45	.84	60	52.15	1.00	95	92.41	1.40
26	21.29	.84	61	53.15	1.00	96	93.84	1.43
27	22.13	.84	62	54.15	1.00	97	95.31	1.47
28	22.98	.85	63	55.16	1.01	98	96.82	1.51
29	23.83	.85	64	56.18	1.02	99	98.38	1.56
30	24.68	.85	65	57.21	1.03	100	100.00	1.62
31	25.53	.85	66	58.24	1.03			
32	26.39	.86	67	59.28	1.04			
33	27.25	.86	68	60.33	1.05			
34	28.11	.86	69	61.38	1.05			

* Calculated by W. Dittmar.

CHANGE OF STANDARD.

To pass from Water of t' to Water of t'' degrees as a Standard, apply the formulæ or tables given on pp. 108, 109, 113. But for $t = 4^\circ, 15^\circ, 15.56^\circ$, the following table is more convenient.

To pass from Water of 15° to Water of 15.56°	To pass from Water of 4° to Water of 15° or 15.56° , add on the correction given for the nearest tabular S.		
	To Water of 15° S=	Correction.	To Water of 15.56° S=
Add on .000 08 to the given value S. The error is less than .000 01	.71 .83 .95 1.07	.000 7 .000 8 .000 9 .001 0	.75 .86 .97 1.08

For the corresponding inverse changes subtract the correction given.

Tables IX., X., XI., are deduced from Mendeleeff's determinations, but not quite directly. Landolt and Börnstein's "*Tabellen*" include (on their page 151), a table which, proceeding from percent to percent, gives the specific gravities $_{15}S_{15}$ to five decimal places. According to its heading the table is "*basirt auf Mendeleeff's Formeln.*" From this table I calculated the entries p of my table IX. to, originally, three decimals, by simple interpolation. At the end, the third decimals in the values p were cut off; and in order to see how this and the neglecting of second differences had affected the results, I calculated the specific gravities for all integer p's from my table as it stood (and now stands in this book), and compared the results with the original specific gravities in L. and B.'s tables. The difference amounted to nil in 51, to ± 1 in 44, and to ± 2 in five cases; Water = 100 000. The values $\frac{\Delta S}{\Delta t}$ I calculated from data found in Mendeleeff's original memoirs in Pogg. Annalen, vol. 138; pp. 103 and 230.

Tables X. and XI. are calculated directly from the entries in L. and B.'s table.

METHYL ALCOHOL.

According to Dittmar and Fawsitt,* the specific gravity of Methyl-Alcohol at temperatures from 0° to 64.7° is in accordance with equation:

$$_4S_t = .810\ 18 - 90.53\ t - .085\ 057\ t^2$$

Their specific gravities for Aqueous Methyl-Alcohols are given in the following table.

* Royal Society of Edinburgh Transactions, vol. 33, part II., p. 523, and pp. 531-533. Year 1886-7.

DITTMAR AND FAWSITT'S TABLE OF THE SPECIFIC GRAVITIES OF
AQUEOUS METHYL-ALCOHOLS at 0° AND AT 15.56° C.; WATER
OF $+4^{\circ} = 100\ 000$.

I.—From 0 to 30 per cent. of CH_3O .

${}_4\text{S}_0 - {}_4\text{S}_t = at + bt^2$; from $t = 0$ to $t = 20^{\circ}$.

Per-centage.	Sp. Gravity at 0° C.	Diff.	a	b	Sp. Gravity at 15.56° C.	Diff.
0	999 87		- 6.0	+ .705	999 07	
1	998 06	- 181	5.4	.694	997 29	- 178
2	996 31	175	4.8	.681	995 54	175
3	994 62	169	3.9	.670	993 82	172
4	992 99	163	3.0	.659	992 14	168
5	991 42	157	- 2.2	.648	990 48	166
6	989 90	152	1.2	.634	988 93	155
7	988 43	147	0.2	.621	987 26	167
8	987 01	142	+ 0.9	.609	985 69	157
9	985 63	138	2.1	.596	984 14	155
10	984 29	134	+ 3.3	.581	982 62	152
11	982 99	130	4.8	.569	981 11	151
12	981 71	128	6.2	.552	979 62	149
13	980 48	123	7.8	.536	978 14	148
14	979 26	122	9.5	.519	976 68	146
15	978 06	120	+ 11.0	.500	975 23	145
16	976 89	117	12.5	.480	973 79	144
17	975 73	116	14.5	.461	972 35	144
18	974 59	114	16.2	.440	970 93	142
19	973 46	113	18.3	.420	969 50	143
20	972 33	113	+ 20.0	.398	968 08	142
21	971 20	113	22.2	.373	966 66	142
22	970 07	113	24.3	.350	965 24	142
23	968 94	113	26.4	.321	963 81	143
24	967 80	114	29.0	.291	962 38	143
25	966 65	115	+ 31.3	.261	960 93	145
26	965 49	116	33.8	.230	959 49	144
27	964 30	119	36.0	.191	958 02	147
28	963 10	120	38.8	.151	956 55	147
29	961 87	123	41.1	.106	955 06	149
30	960 57	130	44.0	.063	953 55	151

II.—From 30 to 100 per cent.

 ${}_4S_0 - {}_4S_t = a t$; from $t = 0$ to $t = 20^\circ$

Per-centage.	Specific Gravity at 0°C.	Diff.	a	Specific Gravity at 15.56°C.	Diff.
30	960 57	- 130	+ 44.36	953 67	
31	959 21	136	45.66	952 11	- 156
32	957 83	138	46.93	950 53	158
33	956 43	140	48.17	948 94	159
34	955 00	143	49.39	947 32	162
35	953 54	146	50.58	945 67	165
36	952 04	150	51.75	943 99	168
37	950 51	153	52.89	942 28	171
38	948 95	156	54.01	940 55	173
39	947 34	161	55.10	938 77	178
40	945 71	163	56.16	936 97	180
41	944 00	171	57.20	935 10	187
42	942 39	161	58.22	933 35	175
43	940 76	163	59.20	931 55	180
44	939 11	165	60.17	929 75	180
45	937 44	167	61.10	927 93	182
46	935 75	169	62.01	926 10	183
47	934 03	172	62.90	924 24	186
48	932 29	174	63.76	922 37	187
49	930 52	177	64.60	920 47	190
50	928 73	179	65.41	918 55	192
51	926 91	182	66.19	916 61	194
52	925 07	184	66.95	914 65	196
53	923 20	187	67.68	912 67	198
54	921 30	190	68.39	910 66	201
55	919 38	192	69.07	908 63	203
56	917 42	196	69.72	906 57	206
57	915 44	198	70.35	904 50	207
58	913 43	201	70.96	902 39	211
59	911 39	204	71.54	900 26	213
60	909 17	222	71.96	897 98	228
61	907 06	211	72.37	895 80	218
62	904 92	214	72.91	893 58	222
63	902 76	216	73.45	891 33	225
64	900 56	220	73.98	889 05	228

II.—From 30 to 100 per cent.—*Continued.*

Per-centage.	Specific Gravity at 0° C.	Diff.	a	Specific Gravity at 15.56° C.	Diff.
65	898 35	- 221	74.51	886 76	- 229
66	896 11	224	75.05	884 43	233
67	893 84	227	75.57	882 08	235
68	891 54	230	76.10	879 70	238
69	889 22	232	76.62	877 14	256
70	886 87	235	77.14	874 87	227
71	884 70	230	77.66	872 62	225
72	882 37	233	78.18	870 21	241
73	880 03	234	78.69	867 79	242
74	877 67	236	79.20	865 35	244
75	875 30	237	79.71	862 90	245
76	872 90	240	80.22	860 42	248
77	870 49	241	80.72	857 93	249
78	868 06	243	81.23	855 42	251
79	865 61	245	81.73	852 90	252
80	863 14	247	82.22	850 35	255
81	860 66	248	82.72	847 79	256
82	858 16	250	83.21	745 21	258
83	855 64	252	83.70	842 62	259
84	853 10	254	84.19	840 01	261
85	850 55	255	84.67	837 38	263
86	847 98	257	85.16	834 73	265
87	845 39	259	85.64	832 07	266
88	842 78	261	86.12	829 38	269
89	840 15	263	86.59	826 68	270
90	837 51	264	87.07	823 96	272
91	834 85	266	87.54	821 23	273
92	832 18	267	88.01	818 49	274
93	829 48	270	88.48	815 72	277
94	826 77	271	88.94	812 93	279
95	824 04	273	89.40	810 13	280
96	821 29	275	89.86	807 31	282
97	818 53	276	90.32	804 48	283
98	815 76	277	90.78	801 64	284
99	812 95	281	91.23	798 76	288
100	810 15	280	91.68	795 89	287

CORRECTIONS OF WEIGHINGS.

(1.) *Absolute Weighings*.—If we want the weight x of an object in terms of a fixed upon unit, for instance in terms of the true Gramme, we need a set of standards, which have their correct weights *in vacuo*, or, at least an ordinary set combined with a correction-table which gives the true weights *in vacuo*. Supposing the weight of the object in air of the density δ * is P Grammes, 1 CC of the object weights S grm., and 1 CC of the *material* which the weight-standards are made of weights S_0 grm., we have for the weights in air the equality :

$$x - \frac{x}{S} \delta = P - \frac{P}{S_0} \delta, \text{ whence}$$

$$x = P \frac{1 - \frac{\delta}{S_0}}{1 - \frac{\delta}{S}}$$

and, as δ is always small compared with either S , we have without noteworthy error

$$x = P \left\{ 1 + \delta \left(\frac{1}{S} - \frac{1}{S_0} \right) \right\} \quad \text{I.}$$

where the second term on the right side is, as a rule, positive. If the very highest precision is not aimed at, we may substitute for S the specific gravity taken in air and in reference to water of any "ordinary" temperature, instead of the corrected specific gravity taken in reference to water of 4° as we ought to.

(2.) *Relative Weighings*.—Only such weighings occur in Chemistry, although our customary mode of stating certain results may involve the naming of absolute units. We speak of the "Weight of one litre of air in grammes," but what we really mean is the specific gravity of the air taken in reference to water of 4° as = 1000. Regnault's constant might have been determined by means of a set of grain-weights.

In Chemistry therefore we are at liberty to choose our own unit of weight, and we may choose as such the actual weight of our (virtual) one-gramme piece in air of the density which prevailed during the set is weighings which enter the respective calculation. It is even more correct to do so than adopt the weight of the "one-gramme piece" *in vacuo*,

* This symbol, in this section, denotes the weight of 1 litre of the air in kilogrammes or of 1 CC in Grammes.

because our sets of weights are adjusted in air. That the value of our unit is strictly speaking variable, is a matter of no real consequence, as a short mental calculation based upon the respective data shows. Nor need we take account of the fact, that, for instance, ten decigrammes made of platinum cannot balance a brass one-gramme piece at *all* air-densities.

Hence, in reducing a weighing to the *vacuum*, we need allow only for the air displaced by the *object*. The above equation then reads,

$$x = P \left(1 + \frac{\delta}{S} \right) \quad \text{II.}$$

In the vast majority of cases the " x " is not determined for its own sake but forms only one term of a ratio $x' : x''$. Taking r as a symbol for the uncorrected ratio, and r_0 as one for the corrected, we have

$$r_0 = \frac{P' \left(1 + \frac{\delta'}{S'} \right)}{P'' \left(1 + \frac{\delta''}{S''} \right)} \quad \text{III.,}$$

$$\text{or,} \quad r_0 = r \left(1 + \frac{\delta'}{S'} - \frac{\delta''}{S''} \right) \quad \text{IIIa.}$$

$$\text{or, if } \delta' = \delta''; \quad r_0 = r \left\{ 1 + \delta \left(\frac{1}{S'} - \frac{1}{S''} \right) \right\} \quad \text{IIIb.}$$

with sufficient exactitude. The more nearly $S' = S''$, the more insignificant the correction. The fact that, in most cases, S' does not differ much from S'' is our apology for (as a rule) omitting the vacuum-correction in ordinary analytical practice.

From Regnault's determination of the weight of 1 litre of air in grammes at 0° and 760 mm. we calculate that the weight of one litre of air of the temperature t and the pressure B is

$$D^* = \frac{.46481 \times B}{273 + t} \quad \text{IV.}$$

Regnault's number however refers to air free of carbonic acid and moisture; if we allow for 0.0004 volumes of CO_2 in ordinary dry air and substitute for 273 the special constant for air, we have, for $B = 760$ mm.,

$$D = \frac{1.294 \times 1.00021 \times 272.4}{272.4 + t} \quad \text{IVa.}$$

* For the gravity at Glasgow.

By means of this formula we have calculated the following table, although we are well aware that the factor 1.000 21, for any purpose, might have been omitted, and 273 have been retained as the reciprocal of the co-efficient of expansion.

TABLE I. WEIGHT, IN GRAMMES, OF 1 LITRE OF (DRY) AIR CONTAINING 0.04 % BY VOLUME OF CARBONIC ACID, supposing the air to be measured at t° C and 760 mm. of mercury of 0° C.

t	Weight of 1 Litre D.	Log D.	t	Weight of 1 Litre D.	Log D.
0°	1.29427	.112025	15°	1.2267	.08875
1°	1.2895	.11043	16°	1.2225	.08724
2°	1.2848	.10885	17°	1.2182	.08573
3°	1.2802	.10727	18°	1.2140	.08424
4°	1.2755	.10569	19°	1.2099	.08274
5°	1.2709	.10413	20°	1.2057	.08126
6°	1.2664	.10256	21°	1.2016	.07977
7°	1.2618	.10101	22°	1.1975	.07829
8°	1.2573	.09945	23°	1.1935	.07682
9°	1.2529	.09791	24°	1.1895	.07535
10°	1.2484	.09637	25°	1.1855	.07389
11°	1.2440	.09483	26°	1.1815	.07243
12°	1.2397	.09330	27°	1.1776	.07098
13°	1.2353	.09178	28°	1.1736	.06953
14°	1.2310	.09026	29°	1.1697	.06809
15°	1.2267	.08875	30°	1.1659	.06665

A far more important correction than that for the carbonic acid is that for the moisture contained in the air in which we are weighing. The hygrometer either gives us the partial tension of the moisture directly or the factor k which tells us what fraction the tension is of the maximum tension p corresponding to the prevailing temperature.

The values p have been determined by Regnault and by Magnus. (A table embodying Regnault's results is given on page 133.) Supposing the tension of the moisture in the air is $=kp$, then we have for the weight of the dry part of our litre of air the expression:

$$\frac{760 - kp}{760} D$$

where D may be taken from the above table, and for the weight of the moisture,

$$k \frac{.03212 \times p \times 9.0024}{273+t} = k \frac{0.28916 p}{273+t} = k "d,"$$

meaning k times the weight of 1 litre of saturated steam of t° . In the following table the values d are calculated for a series of temperatures.

TABLE II. *Weight d of one litre of saturated steam of t° in Grammes.*

t	d	Diff.
0°	.0048	
5°	.0068	.0020
10°	.0093	.0025
15°	.0127	.0034
20°	.0171	.0044
25°	.0228	.0057
30°	.0301	.0073

Although the differences are not constant, if we interpolate in the usual fashion, the result is never wrong by more than 0.2 milligrammes at the outside. Thus, for instance, the d for 27.5° is, by interpolation from the table, 0.02645; by direct calculation, 0.02624; difference = .21 mgrm.

The sum $D \frac{760 - kp}{760} + kd$ gives the weight of 1 litre of air as it stands.

The tables are useful, the first for ordinary work, the second for a preliminary estimation of the effect of the full correction; in actual high-precision practice however it is better to calculate the weight of 1 litre of the air, as (D), by means of the formula.

$$(D) = \frac{.46481 (B - kp)}{273+t} + \frac{.2892 kp}{273+t}$$

$$\text{or } (D) = \frac{1}{273+t} \{ 0.46481 B - 0.17565 kp \} \quad V.$$

where B stands for the reading of the Barometer, reduced to 0° .

REDUCTION OF SPECIFIC GRAVITIES.

SOLIDS.

If a solid weights P grm. in air and P' grm. in water; the specific gravity of the solid is, $S = \frac{P}{P - P'}$ But the determination of P is liable to

the reduction to the vacuum (P' —is not); hence we have for the corrected value (S), if $P - P' = W$,

$$(S) = \frac{P + \delta W}{W + \delta W} \text{ or, dividing above and below by } W,$$

$$(S) = \frac{S + \delta}{1 + \delta} \text{ or } (S) = S + \delta - \delta S - \delta^2; \text{ or, as } \delta^2 \text{ is very small,}$$

$$(S) = S - \delta (S - 1)$$

where δ is the density of the air prevailing at the weighing of the solid. The following table gives the corrections for a series of specific gravities. The correction is taken as the independent variable, and is made to progress by intervals of 0.001. δ is taken = 0.001 226 7, corresponding to $t = 15^\circ$ and $P = 760$ mm.

TABLE III.

S	Correction.	S	Correction.	P.P.	
0.59	+ .000 5	9.97	-.011	ΔS	$\Delta (\text{Corr.})$
1.00	<i>Nil</i>	10.78	.012		
1.41	-.000 5	11.60	.013		
1.82	.001	12.41	.014		
2.63	.002	13.23	.015		
				.01	-.000 012
3.45	.003	14.04	.016	.02	.000 025
4.26	.004	14.86	.017	.03	.000 037
5.08	.005	15.67	.018	.04	.000 049
5.89	.006	16.49	.019	.05	.000 061
6.71	.007	17.30	.020	.06	.000 074
				.07	.000 086
7.52	.008	18.12	.021	.08	.000 098
8.34	.009	18.93	.022	.09	.000 110
9.15	.010	19.75	.023	.10	.000 123
		20.56	.024		
		21.38	.025		

Example.—The specific gravity of a body was determined without correction and found to be = 11.012; what is the correction for the vacuum? Find the nearest tabular S , which is 10.78, apply the correction given for that S (which is -.012) and the result $11.012 - .012 = 11.000$ is right within half a unit of the third place. If the given S was safe to the fourth place (which it rarely is), calculate thus:

To $S = 10.78$ corresponds the correction -.01200
 For $\Delta S = .2$ add -.00025
 For $\Delta S = .032$ add -.00004

$S = 11.01200$ total correction = -.01229

hence $(S) = 10.999\ 71$ or, rather 10.999 7.

Change of Standard.—Supposing the specific gravity of a body is = S , water = 1, what is the specific gravity x in reference to another liquid, whose specific gravity on the water scale is = d ? *Answer:* We have, for the relative weights of equal volumes of

	<i>Water</i>	<i>New Liquid</i>	<i>Body</i>
the values,	1	d	S

Obviously, $x = \frac{S}{d}$ and conversely, if x be given, $S = dx$.

Supposing, for instance, the specific gravity of a salt, taken in reference to alcohol of 0.800 specific gravity had been found = 4.12, the specific gravity in reference to water is = $4.12 \times 0.800 = 3.296$.

In exact specific gravity determinations the temperature of the water, which of course, with the method of immersion, is that of the solid under operation, must be noted. Supposing the specific gravity had been determined at, and consequently in reference to water of t° , what is it in reference to water of t' degrees? Rosetti's table (given on page 107) affords the answer. This table gives the specific gravities of water of 0° , 1° , 2° , &c., in reference to that of water of 4° as = 1; in other words the weight of 1 CC of water of t° . Taking w' w'' w''' , &c. as representing Rosetti's numbers for t° t''° t'''° , &c., we have for the relative weights of equal volumes of

<i>Water of 4°</i>	<i>Water of t°</i>	<i>Body under Exam.</i>
1	w'	x

and supposing the specific gravity of the body in reference to water of t° to have been found = S' , obviously $x = w'S'$; but the same relation holds for any value of t , hence we have, for any two temperatures t' and t'' the equation $w'S' = w''S''$, because either product is the "weight of 1 CC at that temperature." Substituting $1 - \epsilon'$ for w' and $1 - \epsilon''$ for w'' , we have,

$$\frac{S'}{S''} = \frac{1 - \epsilon''}{1 - \epsilon'} = 1 - \epsilon'' + \epsilon' + \text{terms in } \pm \epsilon\epsilon, \epsilon\epsilon\epsilon, \text{ etc.}$$

$$\text{and } \frac{S''}{S'} = \frac{1 - \epsilon'}{1 - \epsilon''} = 1 - \epsilon' + \epsilon'' + \text{terms in } \pm \epsilon\epsilon, \epsilon\epsilon\epsilon, \text{ etc.}$$

where these higher terms may be cancelled, so that we have:

$$S' = S'' + (\epsilon' - \epsilon'') S'', \text{ and}$$

$$S'' = S' - (\epsilon' - \epsilon'') S'$$

Supposing S' in one case, and S'' in another, to have the same value, the correction for passing from t' to t'' is the same numerically, as the one for passing from t'' to t' .

TABLE IV. GIVING THE DENSITY, w , OF WATER AT t° ; THAT AT $4^{\circ} = 1$ ACCORDING TO ROSETTI.

(*Pogg. Annalen ; Ergänzungsband V. p. 268, year 1871*).

t	Density* w	Diff.	t	Density w	Diff.	t	Density w	Diff.
-5°	.999 298		30°	.995 77		65°	.980 74	
-4	455 +157		31	47 -30		66	19 -55	
-3	590 +135		32	17 30		67	.979 64 55	
-2	703 +113		33	.994 85 32		68	08 56	
-1	797 + 94		34	52 33		69	.978 51 57	
0	871 + 74		35	18 34		70	.977 94 57	
+1	928 + 57		36	.993 83 35		71	36 58	
2	969 + 41		37	47 36		72	.976 77 59	
3	991 + 22		38	10 37		73	18 59	
4	1.000 000 +009		39	.992 73 37		74	.975 58 60	
5	.999 990 - 10		40	35 38		75	.974 98 60	
6	970 20		41	.991 97 38		76	38 60	
7	933 37		42	58 39		77	.973 77 61	
8	886 47		43	18 40		78	16 61	
9	824 62		44	.990 78 40		79	.972 55 61	
10	747 77		45	37 41		80	.971 94 61	
11	655 92		46	.989 96 41		81	32 62	
12	549 106		47	54 42		82	.970 70 62	
13	430 119		48	10 44		83	07 63	
14	299 131		49	.988 65 45		84	.969 43 64	
15	160 139		50	19 46		85	.968 79 64	
16	002 158		51	.087 72 47		86	15 64	
17	.998 841 161		52	25 47		87	.967 51 64	
18	654 183		53	.986 77 48		88	.966 87 64	
19	460 194		54	29 48		89	22 65	
20	259 201		55	.985 81 48		90	.965 56 66	
21	047 212		56	34 47		91	.964 90 66	
22	.997 828 219		57	.984 86 48		92	23 67	
23	601 227		58	37 49		93	.963 56 67	
24	367 234		59	.983 88 49		94	.962 88 68	
25	120 247		60	38 50		95	19 69	
26	.996 866 254		61	.982 86 52		96	.961 49 70	
27	603 263		62	34 52		97	.960 79 70	
28	331 272		63	.981 82 52		98	08 71	
29	051 280		64	28 54		99	.959 37 71	
30	.995 765 286		65	.980 74 54		100	.958 66 71	

At 60° Fahr., $w = .999\ 073\ 6$. At 14° Reaumur, $w = .998\ 749\ 5$.

* For most purposes the 6th place may be cancelled. At 15.5°, for instance, $\Delta w = .000\ 005$ corresponds to only $= 0.03^{\circ}$ in temperature.

From Rosetti's numbers we have calculated the specific gravities of water of from 0 to 25° , in reference to water of 15° , ($15.56 = 60^{\circ}$ F.) and ($17.5^{\circ} = 14^{\circ}$ R) respectively, and embodied the results in the following tables, along with Rosetti's own results for water of 0° as a standard.

TABLE V. GIVING THE SPECIFIC GRAVITIES OF WATER OF t° REFERRED TO WATER OF 0° OR 15° AS = 1.

t	0°	15°	Diff.
0°	1.000 000	1.000 712	
1	057	769	+ 57
2	098	810	41
3	120	832	22
4	129	841	9
5	119	831	- 10
6	099	811	20
7	062	774	37
8	015	727	47
9	.999 953	665	62
10	976	588	77
11	784	496	92
12	678	390	106
13	559	271	119
14	429	139	131
15	289	000	140
16	131	.999 842	158
17	.998 970	681	161
18	782	494	188
19	588	299	195
20	388	099	200
21	176	.998 886	213
22	.997 956	667	220
23	730	440	227
24	495	206	234
25	249	.997 959	247

TABLE VI. GIVING THE SPECIFIC GRAVITIES OF WATER OF t° REFERRED TO WATER OF ($15.56^{\circ} = 60^{\circ}$ F.) or ($17.5^{\circ} = 14^{\circ}$ R) AS = 1.

t	15.56°	17.5°	Diff.
0°	1.000 798	1.001 123	
1	855	180	+ 57
2	896	221	41
3	918	243	22
4	927	252	9
5	917	242	- 10
6	897	222	20
7	860	185	37
8	813	138	47
9	751	076	62
10	674	1.000 999	77
11	582	907	92
12	476	800	106
13	357	681	119
14	226	550	131
15	087	411	139
16	.999 928	253	158
17	767	092	161
18	580	.999 904	188
19	386	710	194
20	185	.999 509	201
21	.998 972	297	212
22	753	077	220
23	526	.998 850	227
24	292	616	234
25	044	368	248

With these two tables before one, the reduction of a specific gravity to water 0° , 15° , 15.56° , or 17.5° , becomes very easy. Supposing, for instance, a specific gravity, taken in reference to water of 18° , is $_{18}S$; the specific gravity in reference to water of 15° , is $_{15}S = _{18}S \times .999\ 494 = (1 - .000\ 506) S_{18}$.

For the sake of brevity we will use symbols of the form ${}_bS_a$ for designating the specific gravity of a body at a° , taken in reference to water of b° as = unity.

In the determination of specific gravities 15° and 4° are convenient standard temperatures to adopt, the former for working, the latter for the registration of results. According to Rosetti's table the relation between ${}_4S$ and ${}_{15}S$ is,

$$\begin{aligned} {}_{15}S &= {}_4S + {}_4S \times .000840, \text{ and} \\ {}_4S &= {}_{15}S - {}_{15}S \times 0.000840. \end{aligned}$$

Table (VII.) *To pass from ${}_4S$ to ${}_{15}S$, add } the Correction.*
To pass from ${}_{15}S$ to ${}_4S$ subtract }

S	Correction.	S	Correction.
1.00	.000 84		
1.19	.001	13.10	.011
2.38	.002	14.29	.012
3.57	.003	15.48	.013
4.76	.004	16.67	.014
5.95	.005	17.86	.015
7.14	.006	19.05	.016
8.33	.007	20.24	.017
9.52	.008	21.43	.018
10.71	.009	22.62	.019
11.91	.010	23.81	.020

Example.—Given ${}_{15}S = 20.102$; find ${}_4S$. Adopt the “correction” given for the nearest tabular S , which is 20.24. The “correction” given is .017 hence ${}_4S = 20.102 - .017 = 20.085$, to within $\pm .0005$. If a higher degree of precision is wanted, better calculate the correction by multiplying the given S into 0.000 84. Our Three-Place Logarithm Table No. 2 affords sufficient exactitude.

LIQUIDS.

Suppose the specific gravity of a liquid to be determined, in one case by the plunger-method, and in another by the bottle-method, and the results to be as follows:

(*Plunger.*) The plunger weighs: (1) In air of the density δ , P' gm. (2) in water of the temperature t' , P'' gm., and (3) In the Liquid under examination, at t'' degrees, P''' gm. The uncorrected specific gravity of

the liquid at t'' , taken in reference of water of t' is $\frac{(P' - P'' = L)}{(P' - P'' = W)}$ or $S = \frac{L}{W}$. The result however is liable to corrections: Firstly the displacement in grammes of water must be multiplied with the ratio of the volume of the plunger at t'' to that at t' ; i.e. with the factor $1 + k(t'' - t')$, where k is the co-efficient of expansion of glass, which may be put down at .000 025. We will assume this correction to be included in the W as recorded. In passing to the correction for the displaced air, let us observe that only the weighing in air is liable to the reduction to the vacuum, and accordingly put down the reduced specific gravity (S) as

$$(S) = \frac{L + \delta W}{W + \delta_0 W} = (S_{t'}) \quad \text{I.}$$

We mark the deltas to allow for the possibility of the two displacements being determined at different air-densities.

(*Bottle.*) Let W stand for the weight of water of t' degrees held by a bottle (up to a mark in the neck), the weight being determined in air of the density δ_0 ; L for the weight of liquid of t'' degrees which the bottle holds determined in air of the density δ ; and the uncorrected specific gravity is again $S = \frac{L}{W}$ and the corrections also are the same as those in the case of the Plunger. In both cases the reduced specific gravity is by equation I,

$$(S) = \frac{S + \delta}{1 + \delta_0} = S + \delta - S\delta_0 \quad \text{- Ia.}$$

If the two air-densities are the same, we have

$$(S) = S + \delta(1 - S) = S - \delta(S - 1) \quad \text{- II.}$$

If the highest degree of precision is not aimed at, it may be permissible, instead of determining the air-density, to adopt for it the average value for the place of observation. The following table is calculated on the assumption that the weight of a litre of air is = 1.2267 grm., corresponding to 15° and 760 mm.

Table (VIII.) To reduce a specific gravity to the Vacuum, add on the "Correction."

S	Correction.	S	Correction.
0.65	+ .000 43	1.35	- .000 43
0.70	+ .000 37	1.40	- .000 49
0.75	+ .000 31	1.45	- .000 55
0.80	+ .000 25	1.50	- .000 61
0.85	+ .000 18	1.55	- .000 67
0.90	+ .000 12	1.60	- .000 74
0.95	+ .000 06	1.65	- .000 80
1.00	<i>Nil.</i>	1.70	- .000 86
1.05	- .000 06	1.75	- .000 92
1.10	- .000 12	1.80	- .000 98
1.15	- .000 18	1.85	- .001 04
1.20	- .000 25	1.90	- .001 10
1.25	- .000 31	1.95	- .001 17
1.30	- .000 37	2.00	- .001 23

Supposing the true δ at the time differed from the presumed value 1.2267 by one-tenth of the value, the error in the tabular correction could not amount to more than $\pm .000 123$; we refer to $S = 2$. For any S between 1.0 and 1.05 (which includes all sea-waters) it could not amount to more than .000 006. In other words; very few specific gravity determinations are worth a special determination of the air-density. In all those cases in which an error of half-a-unit of the fourth place can be neglected, the following table will be found convenient, because it calls for no interpolation.

TABLE IX.

S	Correction.
.67	+ .000 4
.76	+ .000 3
.84	+ .000 2
.92	+ .000 1
1.00	<i>Nil.</i>
1.08	- .000 1
1.16	- .000 2
1.24	- .000 3
1.33	- .000 4
1.41	- .000 5
1.49	- .000 6
1.57	- .000 7
1.65	- .000 8
1.73	- .000 9
1.82	- .001 0
1.90	- .001 1
1.98	- .001 2

To reduce a given Specific Gravity to the Vacuum, adopt the correction given opposite the nearest tabular S.

CHANGE OF STANDARD.

All that we said under this head in regard to solids, holds for liquids as well; only in the case of the latter we have to deal with a narrower range of specific gravities, and, in general, are able to attain a higher degree of precision.

By means of Rosetti's table we can at once pass from the specific gravity taken in reference to water of t° to the value holding for water of 4° as a standard. Obviously

$${}_4S = {}_tS \times (\text{the } w \text{ for } t^{\circ}) = {}_tS (1 - \epsilon) \quad - \quad \text{III.}$$

$$\text{and } {}_tS = \frac{S_4}{w} = {}_4S (1 + \epsilon) \text{ practically,} \quad - \quad \text{IIIa.}$$

For a given value S the correction is $\pm S\epsilon$, whether we pass from t° to 4° , or from 4° to t° .

The following table, X, give the negative corrections to be "*added on,*" in passing from 0° , 15° , 15.56° (60° F), 17.5° (14° Reaumur) to water of 4° as a standard, and consequently enables us, by two successive

corrections, to pass, for instance from water of 15° to water of 17.5° as a standard. To pass

TABLE X.

FROM WATER OF 0 TO WATER
OF 4°, add on

S	Correction.
0.7	<i>Minus</i> .000 090
0.8 - -	.000 103
0.9 - -	.000 116
1.0 - - -	.000 129
1.1 - - -	.000 142
1.2 - - -	.000 155
1.3 - - -	.000 168
1.4 - - -	.000 181
1.5 - - -	.000 194
1.6 - - -	.000 206
1.7 - - -	.000 219
1.8 - - -	.000 232
1.9 - - -	.000 245
2.0 - - -	.000 258

ΔS	Δ (Corr.)
1	.000 129
.007 75	.000 001

FROM WATER OF 15.56 TO WATER
OF 4°, add on

S	Correction.
0.7	<i>Minus</i> .000 648
0.8	.000 741
0.9 - -	.000 834
1.0 - -	.000 926
1.1 - -	.001 019
1.2 - - -	.001 112
1.3 - - -	.001 204
1.4 - - -	.001 297
1.5 - - -	.001 390
1.6 - - -	.001 482
1.7 - - -	.001 575
1.8 - - -	.001 668
1.9 - - -	.001 760
2.0 - - -	.001 853

ΔS	Δ (Corr.)
1	.000 926
.001 079	.000 001

FROM WATER OF 15 TO WATER
OF 4°, add on

S	Correction.
0.7 - - -	<i>Minus</i> .000 588
0.8 - - -	.000 672
0.9 - - -	.000 756
1.0 - - -	.000 840
1.1 - - -	.000 924
1.2 - - -	.001 008
1.3 - - -	.001 092
1.4 - - -	.001 176
1.5 - - -	.001 260
1.6 - - -	.001 344
1.7 - - -	.001 428
1.8 - - -	.001 512
1.9 - - -	.001 596
2.0 - - -	.001 680

ΔS	Δ (Corr.)
1	.000 840
.001 19	.000 001

FROM WATER OF 17.5 TO WATER
OF 4°, add on

S	Correction.
0.7	<i>Minus</i> .000 875
0.8 - -	.001 000
0.9 - - -	.001 125
1.0 - - -	.001 250
1.1 - - -	.001 376
1.2 - - -	.001 501
1.3 - - -	.001 626
1.4 - - -	.001 751
1.5 - - -	.001 876
1.6 - - -	.002 001
1.7 - - -	.002 126
1.8 - - -	.002 251
1.9 - - -	.002 376
2.0 - - -	.002 501

ΔS	Δ (Corr.)
1	.001 250
.000 800	.000 001

NOTE.—A glance at the appendices to the tables shows that, in interpolating even with the 17.5° table we need not take in more than the first 3 decimals in the given specific gravity, even if we wish to retain the 6th place in the calculation. If we wish to calculate to the 5th place, two places in the given S are sufficient to find the true correction.

Example.—The specific gravity of a liquid, taken with reference to water of 17.5° is 1.5072; what is it with reference to water of 0° ? Answer. To pass to 4° , we subtract, by table, .001 88; to pass then to water of 0° , we add on .000 19; total correction = $-.001\ 88 + .000\ 19 = -.001\ 69$. Hence Corrected Specific Gravity = 1.50551, or rather 1.5055, if the given value was safe to only the fourth place.

CORRECTION OF A SPECIFIC GRAVITY FOR THE THERMIC EXPANSION OF THE BODY UNDER EXAMINATION.

To effect such a correction with rigorous exactitude, we must have a table for the body corresponding to that of Rosetti for water, or an equivalent formula. Neither of the two as a rule exists. In cases however, where the difference of temperature to be allowed for is small and the highest degree of precision is not demanded, the following rules may be followed.

Solids.—Supposing the co-efficient of cubic expansion, reckoned from 0° upwards, is k , and the specific gravity at t' degrees (*i.e.*, the weight of one volume, taken in reference to that of 1 vol. of water of standard temperature as = 1) is = S' , we have for the two specific gravities S' and S'' at t' and t'' respectively,

$$S'' = S' \frac{1 + kt'}{1 + kt''} \text{ or } S'' = S' [1 - k(t'' - t')] \text{ practically.}$$

Liquids.—In these, to express the relation between the volume at t' and the volume at t'' , it generally takes a function of more than the first degree; we have, in general,

$$v'' = v' [1 + a(t'' - t') + b(t'' - t')^2 + c(t'' - t')^3] +$$

but, as a matter of experience, if we take the specific gravity at t'' as being $S'' = S' [1 - \text{const.}(t'' - t')]$, the approximation reached is far better than would be afforded by the corresponding linear function for the relation between v'' and v' .

To determine the constant, we need only two determinations of the specific gravity, one at t' and another at (say) $t' + 10^\circ$. By means of the constant, then, the S for any intermediate t is easily calculated.

THE AUTHOR'S DIFFERENTIAL METHOD OF SPECIFIC GRAVITY
DETERMINATION.

The method is intended for the determination of the difference of specific gravity between, for instance, water and a sea-water or other natural water; a standard acid, *known* to contain 49 grammes of real H_2SO_4 per litre, and one only *supposed* to contain this proportion, between real absolute alcohol, and an apology for the preparation, etc., etc. It is as follows:—A test-tube shaped pycnometer, provided with a perforated glass stopper, is charged with Liquid I. and suspended within a mass of the same liquid contained in a cylinder by means of a thin platinum wire. As soon as perfect equilibrium of temperature is established, the effective weight of what hangs at the balance is determined; let this weight be $=P_1$ gm. The pycnometer is now emptied into a beaker, charged with Liquid II. at the same temperature, again suspended from the balance and the apparent weight of the whole in its now condition determined. Let this second weight be $=P_2$. If the temperature remained rigorously constant at t' degrees, the difference sought is easily calculated. Taking the volume of 1 gm. of water of t' degrees as unit-volume, and V as a symbol for the capacity of the pycnometer at t' degrees, we have

$$VS_2 + \text{constant} = P_2$$

$$VS_1 + \text{constant} = P_1$$

$$\text{hence} \quad S_2 - S_1 = \frac{P_2 - P_1}{V} \quad \text{I.}$$

If the temperature between the determinations of P_1 and that of P_2 rises from t' to t'' degrees, the equation no longer remains true. Supposing the specific gravities of the two liquids pass from S_1' and S_2' to S_1'' and S_2'' ; the true weight of the pycnometer $=G$ gm., and the weight of unit-volume of the glass $=s'$ gm. at t' degrees, we have

$$(1) \quad G - \frac{G}{s'} S_1' + (\text{constants} = C) = P_1$$

$$(2) \quad G - \frac{G [1 + k(t'' - t')]}{s'} S_1'' + C + V [1 + k(t'' - t')] [S_2'' - S_1''] = P_2$$

If we know the thermic coefficient λ of change of specific gravity for Liquid I., *i.e.*, if we know that $S_1'' = S_1' [1 - \lambda(t'' - t')]$, the second term in equation (2) becomes

$$- \frac{G S_1'}{s'} [1 - (t'' - t') (\lambda - k)]$$

By effecting this substitution and subtracting equation (1) from equation (2) we have,

$$P_2 - P_1 = \underbrace{\frac{G S'_1}{s'} (t'' - t') (\lambda - k) + V [1 + k (t'' - t')] [S_2'' - S_1'']}_{*}$$

* In this term, $\frac{G S'_1}{s'}$ is obviously the weight of Liquid I. displaced by the *glass* of the pycnometer at t' degrees; and the whole of the term is a very small quantity, which we will call q . We then have:

$$S_2'' - S_1'' = \frac{P_2 - P_1 - q}{V [1 + k (t'' - t')]} \quad \text{III.}$$

k , of course, stands for the cubic coefficient of expansion of the glass.

There is however no need of determining the values λ and k , and calculating q . Supposing P_1 to have been determined at t' degrees, and the temperature to show a tendency to rise, raise the temperature by means of a water-bath to, say, a whole degree above t' , determine the value which P_1 assumes at this higher temperature, and thus place yourself in a position to calculate the correct P_1 for any intermediate temperature. Then re-establish, by intention, the original temperature, determine the exact temperature, and the correct P_2 corresponding to it; combine this with the corresponding P_1 , and calculate by equation I.

CALIBRATION.

In the designing of volumetric instruments the following geometric formulæ may be useful:—

The volume of a sphere of the diameter d is

$$V = \frac{4}{3} \left(\frac{d}{2} \right)^3 \pi = .5236 d^3 \quad \text{I.}$$

The volume of a cylinder of the diameter d and the height h is

$$V = \left(\frac{d}{2} \right)^2 \pi h = .7854 d^2 h \quad \text{II.}$$

Applications.—(1) What must be the inner diameter of a spherical pipette-bulb, if it is to hold V_{cc} ? *Answer:* by I.,

$$d = 1.2407 \sqrt[3]{V} \text{ centimetres.}$$

(2) What must be the inner width of a glass tube intended to be made into a Mohr's burette, if every CC-division is to be n mm. long? *Answer:* by II.,

d must satisfy the equation: $0.7854 d^2 n = 1000$ cubic millimetres,

$$\text{hence } d = 35.68 \frac{1}{\sqrt{n}} \text{ millimetres.}$$

For calculations like these, three-place logarithms afford amply sufficient precision.

Example: To find the d of a sphere whose $V = 120$ cc. $\log. 120 = 2.079$; $\log.$ of cube-root $= \frac{1}{3}$ of $2.079 = 0.693$; $\log. 1.241 = 0.094$, hence $\log. d = .094 + 0.693 = .787$; $\text{num.} = 6.12$ centimetres.

The following Table will be found useful in designing dilatometers, graduated pipettes, etc. :—

RELATION BETWEEN THE BORE (d Mm.) OF A GLASS TUBE AND ITS CAPACITY, C ,

In Cubic Millimetres or Milligrammes of Water, per 1 Mm. of Length.

d	C	d	C	d	C	d	C	d	C
1	.785	2.2	3.801	3.4	9.079	4.6	16.62	9.0	63.62
1.1	.950	2.3	4.155	3.5	9.621	4.7	17.35	10	78.54
1.2	1.131	2.4	4.524	3.6	10.18	4.8	18.10	11	95.0
1.3	1.327	2.5	4.909	3.7	10.75	4.9	18.86	12	113.1
1.4	1.540	2.6	5.309	3.8	11.34	5.0	19.63	13	132.7
1.5	1.767	2.7	5.726	3.9	11.95	5.5	23.76	14	153.9
1.6	2.011	2.8	6.158	4.0	12.57	6.0	28.27	15	176.7
1.7	2.270	2.9	6.605	4.1	13.20	6.5	33.18	16	201.1
1.8	2.545	3.0	7.069	4.2	13.85	7.0	38.49	17	227.0
1.9	2.835	3.1	7.548	4.3	14.52	7.5	44.18	18	254.5
2.0	3.142	3.2	8.043	4.4	15.21	8.0	50.27	19	283.5
2.1	3.464	3.3	8.553	4.5	15.90	8.5	56.75	20	314.2

GRADUATION OF EUDIOMETERS,

meaning tubes intended for the measurement of gases over mercury.

The system usually followed is to lay on a millimetre-scale, and then to determine the volume-values of the several marks, in terms of an arbitrary unit, by either gravimetric or volumetric calibration. In the case of ordinary (open) eudiometers we usually resort to the

Volumetric Method.—The eudiometer having been fixed in a vertical position, mouth upwards, we introduce small successive exactly equal quantities of mercury, and, after each addition of metal, take the position of the top of the meniscus in reference to the scale. If the readings were $r_1 r_2 r_3 r_4$, the corresponding mercury-volumes are $1 v, 2 v, 3 v, 4 v$, etc., where v stands for the capacity of our standard mercury-measure. But for this v we may select any number we please, and the best number to select is the average number of millimetres which one fill of mercury corresponds to in the cylindrical part of the tube. We then, if the zero of the scale is in its proper place, have approximately $(nv = V) = r$; and in a practical sense exactly, $\Delta V = \text{constant } \Delta r$, for any small difference of level Δr . But the *gas-volume* corresponding to r is obviously greater than the mercury-volume up to r (at that stage of the calibration) by twice the empty space between the surface of the mercury and the horizontal plane passing through r . To measure this space, we bring it into the shape of a cylindrical disc by causing the meniscus to flatten out into a plane, and take the reading of this plane. If it stands say δ mm. below r , the volume of the empty space is (and before was) $= \delta$ units, and the correct gas-volume corresponding to r_n is $nv + 2\delta$. From the true gas-volumes for $r_1 r_2 r_3 r_4 \dots$ we calculate those corresponding to all the several millimetre-strokes, on the assumption, that within each interval, the volume-value of 1 mm. is a constant, and enter the results into a "*calibration-table*."

The Gravimetric Method must be resorted to, if we have to deal with the measuring tube of any of the so-called "*manometric*" gas-apparatus. It consists in this, that after having filled the tube completely with mercury, we let out metal, first down to, say, $r = 20$, then down to 40, 60, 80, etc. (about), and, after each such discharge, take the *exact* reading of the meniscus and the exact weight of mercury let out so far. The rest requires no explanation. In the course of a calibration the temperature must be kept constant to within one or two degrees, and it is as well to supplement the calibration-table by the statement of the cubic-centimetre-value of what, for the respective instrument, is the unit of volume. (See below.)

TITRIMETRIC APPARATUS.

These are customarily graduated so that the reading gives the respective volume directly. Titrimetric apparatus must be calibrated gravimetrically with water, and this in a manner which goes strictly parallel to the way in which the instrument is meant to be used. A 50 CC pipette for instance can be expected to deliver *this exact volume* of liquid only if it be emptied out in a certain *prescribed manner*. The same principle must guide us in the testing of ready-made apparatus.

To test a Burette (which we will assume to be divided into 100 "degrees" of unstated magnitude) we, first of all, procure a supply of distilled water of very nearly the temperature of the balance-room. With such water we fill the burette up to the zero-mark, we then empty it, successively, down to (say) 10, 20, 30, . . . 100 degrees, into a tared flask, and after each delivery, determine the weight of water let out so far. The several water-weights $w_1 w_2 w_3 w_4 \dots$ ought to be proportional to the corresponding nominal volumes $n_1 n_2 n_3 n_4 \dots$, but we must allow for experimental errors, and all we can expect is, that it is possible to find a number C, which, if taken as designating the water-value of one degree, reduces the errors of the several marks, *i.e.*, the differences

$$n_1 C - w_1 = \delta_1; n_2 C - w_2 = \delta_2; n_3 C - w_3 = \delta_3 \dots \text{III.}$$

to a sufficient degree of smallness. To test the burette in this sense, a practically sufficient method is to calculate C from the largest water-weight, and the corresponding nominal value, or, what is better, to calculate C from (say) the experiments for $n = 80, 90$, and 100 , to take the mean C_0 of the three results, and to (mentally or actually) establish the above equations III. with C_0 substituted for C.

If the differences δ are less than the corresponding "*tolerations*," $\Delta_1 \Delta_2 \Delta_3 \dots$, the burette may pass. But these differences may over-rate the errors of the instrument as one designed for relative measuring. We do better justice to the burette and to our own work, if we calculate by the formula :

$$C = \frac{w_1 + w_2 + w_3 + \dots}{n_1 + n_2 + n_3 + \dots} \quad \text{IV.}$$

The C found by this equation reduces the algebraic sum of the errors of the several nominal water-weights, *i.e.*, the sum of the quantities

$$Cn_1 - w_1 = \delta_1; Cn_2 - w_2 = \delta_2; Cn_3 - w_3 = \delta_3 \dots$$

to nothing.

A still better method is, so to choose C, as to reduce the sum of the

squares of the ratios $\frac{\delta}{\Delta}$ to the minimum. For this purpose we form the equations

$$\frac{Cn_1 - w_1}{\Delta_1} = \epsilon_1; \quad \frac{Cn_2 - w_2}{\Delta_2} = \epsilon_2; \quad \frac{Cn_3 - w_3}{\Delta_3} = \epsilon_3 \dots V.,$$

raise both sides of each to the power 2, sum up the expressions $\left(\frac{Cn - w}{\Delta}\right)^2$, differentiate in respect to C, and equate the differential coefficient $\frac{d(\sum \epsilon^2)}{dC}$ with nothing. The practical rule is, to multiply the several expressions $\frac{Cn - w}{\Delta}$, each with the coefficient of its term in C, to take the sum of the expressions as = 0, and solve this equation; thus:

$$\left\{ \begin{aligned} &\left(\frac{n_1}{\Delta_1} \right)^2 C - \frac{n_1}{(\Delta_1)^2} w_1 \\ &+ \left(\frac{n_2}{\Delta_2} \right)^2 C - \frac{n_2}{(\Delta_2)^2} w_2 \\ &+ \left(\frac{n_3}{\Delta_3} \right)^2 C - \frac{n_3}{(\Delta_3)^2} w_3 \end{aligned} \right\} = 0 \quad \text{VI.}$$

$$C = \frac{\sum \left(\frac{n}{\Delta^2} w \right)}{\sum \left(\frac{n}{\Delta} \right)^2} \quad \text{VIa.}$$

It is hardly necessary for us to say that the third of our methods of calculation was developed only for the information of the *student*; in practice the second, and even the first method suffices. If we have to deal with a

Set of Measures, intended to be used as, so to say, *one* apparatus, we examine the several individual measures in the manner explained for the case of a burette; only, as it is impossible, in a protracted series of testings, to maintain a constant temperature, we must take care, if necessary, to reduce the several water-weights all to the same temperature t_0 ; *i.e.*, from the weight of water of t° , to calculate the weight of water of t_0 degrees which would fill the same space; the expansion of the glass may be neglected. If we adopt 15° as our standard-temperature, the Table, given on page 123, will be found more

convenient than Rosetti's. Before proceeding to calculate the unit C_0 for the whole set, we compare the several values $C_1 C_2 C_3$ found for the individual instruments (allowing for the uncertainties in the corresponding water-weights), and, if it should happen that one or more of these differ considerably from the rest, we, of course, reject the respective apparatus. What we have to do next depends on the degree of closeness with which the numbers C of the apparatus retained agree with one another. To simplify matters, we will assume that we had to deal with a set in which the *Fluidgramme** is the intentional unit. In this case we, of course, reduce all the water-weights to fluidgrammes, and a glance at the results may enable us to at once pass the set as sufficiently exact. Should the agreements between the true and the nominal volumes not be *quite* what we expected, it is worth while to calculate a unit for the set by means of equation IV., which, in this case, assumes a very simple form. Taking W as designating the true volume corresponding to a given apparatus, in fluidgrammes, and n for the nominal volume, we always have $W - n = d$, where d is a small number, compared with W . Hence we have $C_0 = \frac{n_1 + d_1 + n_2 + d_2 + n_3 + d_3 + \dots}{n_1 + n_2 + n_3 + \dots}$ or

$$C_0 = 1 + \frac{d_1 + d_2 + d_3 + \dots}{n_1 + n_2 + n_3 + \dots} \quad \text{VII.}$$

which expression is easily computed.

The value C_0 having been computed, we multiply it with 10, 50, 100, 500, etc., to find the capacities which (say) the 10 CC-pipette, the 50 CC-burette, the 100 CC-flask, the "half-litre-flask," etc., ought to possess, and compare the results with the actual capacities. The differences are the errors of the several apparatus in their capacity as measures based upon the adopted unit, C_0 fgrm. If the result is still unsatisfactory the set must be rejected, unless it should happen that all the burettes and graduated pipettes, if taken as a set for themselves, prove up to the mark, in which case we may adopt *their* constant as *the* constant for the whole set, calculate the capacities which the several one-mark measures ought to have, and in each case put on a correct mark in addition to the original one.

GAUGING IN ABSOLUTE UNITS.

(a) *With Water.*

Rosetti's Table (page 107) gives, for any of the temperatures we need care for, the weight in grammes of one cubic centimetre of water; as a number w conveniently translatable into $1 - \epsilon$. Up to 25° the values ϵ

* See page 71 under "Metric System."

are all less than .003 hence, as the reciprocal of $1 - \epsilon$ is in any case $= 1 + \epsilon + \epsilon^2 + \epsilon^3 \dots$ (if we can afford to neglect an error of .000 009, and as a rule we can), the volume of one gramme in cubic centimetres, up to 25° at least, may be put down as $1 +$ the respective ϵ . To give an example: At 16° Rosetti's entry for w is .999 002, hence $\epsilon = .000\ 998$, and the volume of 1 gramme = 1.000 998 CC. And, if we find it convenient to take water of t° instead of water of 4° , as our standard, we need only add the ϵ for t° to the several entries of Rosetti's Table to find the specific gravities of water in terms of our chosen standard. And, supposing the results are $1 + \delta_1$, $1 + \delta_2$, $1 + \delta_3 \dots$ (for t' , t'' , t''' , . . .), the corresponding relative volumes of unit weight are $1 - \delta_1$, $1 - \delta_2$, $1 - \delta_3 \dots$ respectively. Thus, for instance, since

In passing to Water of 15° as our standard, we have with sufficient exactitude,

For $t =$	4°	10°	15°	20°
$W_4 =$	1	.999 75	.999 16	.998 26
Spec. Gr.	1.000 84	1.000 59	1	.999 10
Rel. Vol.*	.999 16	.999 41	1	1.000 90

In determining the capacity of a vessel in real cubic centimetres or litres we must not forget to reduce the water-weight to the vacuum, before multiplying with $1 + \epsilon$.

To pass from a given volume *in cubic centimetres* to that in fluid-grammes or cubic inches, multiply with the following factors:—

Wanted.	Factor f .	Logarithm f .
Fluidgrammes,	.998 081	.999 166 - 1
Cubic Inches,	.061 027 1	.785 522 - 2

In the latter case, of course, if we want an exact result, the volume in CC must be based upon a weighing effected with the true gramme; the same, indeed, must be the case if we want the volume in *true* cubic centimetres. In chemistry, however, we never do; the so-called "*cubic centimetre*" is only a misnomer for *milli-litre*, and "*litre*" with the chemist means the volume, at 4° , of 1000 of his "*grammes*" of water. Perhaps it is not quite superfluous to add that those who are in the habit of using the grain and fluid-grain as their unit, may view their grain as a "*gramme*" and use our formulae and tables made out in terms of

* Or, *specific gravity* of water of 15° , taking that of water of 4° , 10° , 20° respectively as = 1. Compare Supplementary Tables to Rosetti's, page 108.

(A glance at these numbers shows, that in the calibration of the smaller titrimetric apparatus, such as pipettes up to 20 CC, a variation of even $\pm 5^\circ$ in the temperature of the water may be neglected without adding to the unavoidable errors.)

grammes, fluidgrammes, milli-litres; the "litre" of course they must define as the volume, at 4°, of 1000 true grains of water, "true" referring to the vacuum-correction. Whenever the highest degree of precision is *not* demanded, the density of the air needs not be expressly determined, and we may save ourselves some trouble by using the following Table:—*

A QUANTITY OF WATER WHOSE UNCORRECTED WEIGHT, DETERMINED WITH BRASS STANDARDS, IN AIR OF t° AND 760 MM. PRESSURE, IS 1000 GRAMMES, OCCUPIES AT t° THE VOLUME OF $1000 + x$ FLUIDGRAMMES = $1000 + y$ MILLI-LITRES.

t	x	y All positive.	t	x	y All positive.
0°	-.65	+ 1.27	15°	nil.	+ 1.923
4	.79	1.12	16	+.15	2.08
8	.70	1.22	17	.31	2.23
9	.64	1.28	18	.49	2.42
10	.57	1.35	19	.68	2.61
11	.48	1.44	20	.88	2.81
12	.38	1.54	21	1.09	3.02
13	.27	1.66	22	1.31	3.24
14	.14	1.78	23	1.53	3.46
15	nil.	1.923	24	1.77	3.69

If the water was weighed in air of (t° and) $760 \pm b$ mm., the values y , strictly speaking, ought to be corrected by adding on (in the sense of algebra), $\pm D \frac{.8795 \times b}{760}$ where D is the weight of one litre of air in grammes at t° and 760 mm. and .8795 is the difference between one litre and the volume of one kilo of brass. Between 10° and 24° the mean value for D is 1.2190; adopting this value for all temperatures from 10° to 24°, we have for the correction $\pm .0014 b$.

Example: At 15° and 760 mm. the 1000 grm. of water (uncorrected weight) occupy 1001.923 CC. Hence at 15° and $760 - 30 = 730$ mm.,

* This is not a mere transcription of the Table which I calculated for, and published in, my article "Analyse volumetrische" in Fehling's Handwörterbuch and subsequently incorporated in the second edition of this book. The Table was re-calculated for the present edition on the basis of *Rosetti's* Table for the specific gravities of water. (The older Table had been based on some other table taken from I forget which handbook.) The differences, as a comparison will show, amount sometimes to several units of the second place; yet, in opposition to all ordinary applications, they may be called insignificant.

the volume is $1001.923 - (30 \times .00141) = 1001.923 - .042 = 1001.881$ CC, $= 1000 - .042$ fluidgrammes. The correction, as we see, does not come to much; in the calibration of a litre-flask at least it falls within the observational errors.

(b) *With Mercury.*

According to Volkmann's calculations (Pogg. Annalen, vol. 13, p. 209, year 1881), 1 CC of mercury of 0° weighs 13.5953 grammes. The apparent weight in air of 15° and 760 mm. is 13.59608,* if determined with brass weights. Log.* = 1.133 414. Deviations from the air-density referred to may be neglected. Hence, in reference to uncorrected weight thus determined, we may say: 1 gramme of mercury, at t° , occupies $\frac{1}{13.59608} (1 + kt)$ cubic centimetres; and 1 CC of mercury of t° weighs $13.59608 \div (1 + kt)$ grammes. k stands for the mean coefficient of expansion of mercury, from 0° to t° . According to Levi, k , from 0° to 30° , has the practically constant value of .000 181 43. From Levi's numbers we have calculated the following Table:—

TABLE GIVING THE VOLUMES V, IN MILLI-LITRES, OF 1000 GRMS.
OF MERCURY (UNCORRECTED WEIGHT), AT ALL TEMPS.,
FROM 0° TO 30°.

t	V	Log. V.		
0°	73.5506	.866 586	Partes Prop. for the V.	
1	.5639	665		
2	.5773	744		
3	.5906	822		
4	.6040	901		
5	73.6173	.866 980	Δt	ΔV
6	.6306	.867 058	0.1	.001 33
7	.6440	137	.2	.002 67
8	.6573	216	.3	.004 00
9	.6707	295	.4	.005 34
10	73.6840	.867 373	0.5	.006 67
11	.6973	452	.6	.008 00
12	.7107	531	.7	.009 34
13	.7240	609	.8	.010 67
14	.7374	688	.9	.012 01
15	73.7507	.867 766	1.0	.013 34
16	.7640	845	For the Logs. V.	
17	.7774	923		
18	.7907	.868 002		
19	.8041	080		
20	73.8174	.868 159	Δt	$\Delta (\log. V.)$
21	.8308	238	0.1°	VI. 7.9
22	.8441	316	.2	15.7
23	.8575	395	.3	23.6
24	.8708	473	.4	31.4
25	73.8842	.868 552	0.5	39.3
26	.8975	630	.6	47.2
27	.9109	709	.7	55.0
28	.9242	787	.8	62.9
29	.9376	865	.9	70.7
30	73.9509	.868 944	1.0	78.6

To find the weight of 1 milli-litre of mercury of t° in grammes, take the dekadic complement of Log. V for that t , and find the number.

Example: For $t = 30^\circ$, Log. V = .868 944; dek. compl. = .131 056, num. = 13.5225, which is the weight (in air) of 1 CC in grammes.

The capacity of a vessel of course increases when the temperature increases, in accordance with equation :

$$V' = V[1 + \gamma (t' - t)] \quad \text{IX.}$$

where V stands for the capacity at t and V' for the capacity at t' degrees, and γ for the coefficient of cubic expansion of the respective material. In the case of *Glass*, γ , for ordinary purposes,* may be put down at 0.000 025, *i.e.*, we may assume that any glass vessel between 0° and say 100° , expands by 0.25 volumes per 1000 volumes per 10° , which at once shows that, in titrimetric and gas-analysis, the capacities of our measures, as determined by calibration at, say 15° , may be assumed, in general, to hold from 5° to 25° , *i.e.*, for all ordinary temperatures without adding appreciably to the unavoidable uncertainty of the determinations. The correction, in ordinary laboratory practice, comes in only, when a gas has to be measured at artificially established high temperatures. For an example, suppose a eudiometer, calibrated at 15° , had been used for measuring a gas at 100° . If the volume, as taken from the Calibration Table, is V , the true volume at 100° is (V) , and

$$(V) = V(1 + 85 \times 0.000\ 025) = 1.002\ 13\ V. \quad \text{X.}$$

(For the necessary corrections of the observed pressure see under "Gasometry.")

Recknagel (Pogg. Ann. 1864, volume 123, p. 115; we copy from Kraut-Naumann, Physikalische Chemie, Heidelberg, 1877, p. 414), in the course of a research on thermometry, arrived at the following values for the mean coefficient of cubic expansion of "ordinary" German glass between 0° and the following temperatures t :—

t	Coefficient.
10°	.000 0255
20	0258
30	0260
40	0262
50	0265
60	0267
70	0269
80	0272
90	0274
100	0276
150	0288
200	0299

* For high-class precision work, its value must be determined specially for the given vessel, because each kind of glass has its own coefficient of expansion.

GASOMETRY.

Volumetric gasometry is based upon the following propositions:—

(1) *Concerning Gases generally.* A given quantum of any gas, at a given temperature and pressure, can have only *one* volume, and, at a given temperature and volume, can exhibit only *one* pressure.

(2) *Concerning Perfect Gases.* If two perfect gases be placed under equivalent conditions of temperature and pressure,* their consequent volumes V' and V'' bear to each other a constant ratio, independent of t and P .

The gas to be measured is, as a rule, shut up over a continuous mass of mercury which is in touch with both the gas and the atmosphere, and there generally is a column of mercury, h mm. high, which (a) presses with the gas against the atmosphere, or (b) with the atmosphere against the gas. In case (a) the tension of the gas which balances the net pressure which it is under, is $P = B - h$; in case (b) we have $P = B + h$, where B stands for the height of the column of mercury in the barometer. Our statement includes that in gasometry we take the pressure of a column of mercury 1 mm. high as our unit for P . The unit, as thus defined, is variable, but in the majority of cases we have to deal only with the comparative measurement of a series of gases all effected at the "ordinary," meaning at very nearly the same, temperature, and the unit may be assumed to be constant without adding to the unavoidable errors. (In regard to absolute measurement of gas-pressure see below.)

If the gas is shut up over *water*, or perhaps a combination of columns of different liquids, the heights of these must be "*reduced*" to mercury.

Two columns of liquid, of which one consists of a Liquid I., whose specific gravity is S' , and the other of a Liquid II., of the specific gravity S'' , exert the same pressure, if their heights are so adjusted, that

$$H'S' = H''S'' \quad \text{I.}$$

$$\text{or if } H' = \frac{H''S''}{S'} \text{ or if } H'' = \frac{H'S'}{S''} \quad \text{Ia.}$$

The values S' and S'' ought to, but in practice need not, be reduced to the vacuum. If Liquid I. is water of t° and Liquid II. mercury of t' degrees, and the values S be referred to water of 4° as $= 1$, we have

$$HS' = \frac{13.5953 h}{(1 + kt')}$$

* Meaning: "be at the same disgregation;" *vide infra*.

where $S' = w'$, meaning the weight of 1 CC of water at t' degrees, as given in Rosetti's Table (see page 107); 13.5953 is the weight of 1 CC of mercury at 0° , and k the coefficient of expansion of mercury from 0° to t' degrees. If $t' = 15^\circ$, we have $S' = .999\ 160$; k , between 0° and 30° , is .000 18143, whence $1 + 15k = 1.002\ 7215$. Hence,

$$H = \frac{13.5953\ h}{1.002\ 7215 \times .999\ 160}, \text{ or}$$

$$H = 13.56980\ h; \quad \text{Log.} = 1.132\ 574 \quad \text{II.}$$

$$h = .073\ 6931\ H; \quad \text{Log.} = .867\ 426 - 2 \quad \text{III.}$$

The following Table is calculated from eq. III. This Table, though strictly correct for 15° only, may, for all ordinary purposes, be used for any temperature from, say, 10° to 20° at least.

TABLE I. FOR THE REDUCTION OF WATER PRESSURE TO MERCURY PRESSURE, AT 15°C .

H Mm. inches, etc., of Water are equivalent to h Mm. inches, etc., of Mercury.

H	h	H	h	H	h	H	h	H	h
0	0.00	0	0.00	100	7.37	200	14.74	300	22.11
1	.07	10	0.74	110	8.11	210	15.48	400	29.48
2	.15	20	1.47	120	8.84	220	16.21	500	36.85
3	.22	30	2.21	130	9.58	230	16.95	600	44.22
4	.29	40	2.95	140	10.32	240	17.69	700	51.59
5	.37	50	3.68	150	11.05	250	18.42	800	58.95
6	.44	60	4.42	160	11.79	260	19.16	900	66.32
7	.52	70	5.16	170	12.53	270	19.90	1000	73.69
8	.59	80	5.90	180	13.26	280	20.63		
9	.66	90	6.63	190	14.00	290	21.37		

For the Reduction of Mercury Pressure to Water Pressure at 15° calculate by eq. II., using logarithms, or else use the following multiples of the constant, here rounded off to 13.57 :—

h =	1	2	3	4	5	6	7	8	9
H =	13.57	27.14	40.71	54.28	67.85	81.42	94.99	108.56	122.13

Absolute Measurement of Pressures.

For these we adopt as unit of pressure or tension, the pressure exerted by a column of mercury of 0° and of the height of 1 true millimetre at a latitude of 45° and sea-level. Hence supposing we find the

pressure of a gas to be equal to "P mm." of mercury of t° , to refer it to our absolute unit, we must apply the following corrections:—

(1) We must reduce our "P mm." to true milli-metres. If our scale is, as usual, laid down on glass, and has its true length at t_0 degrees, the P mm. divisions, at t° , are (P) mm. long, and

$$(P) = P [1 + \lambda (t - t_0)], \text{ if } t > t_0.$$

$$(P) = P [1 - \lambda (t_0 - t)], \text{ if } t_0 > t.$$

$\lambda = \frac{1}{3} \gamma$ is the coefficient of linear expansion of the respective kind of glass, which, however, for ordinary purposes, needs not be determined expressly but put down at

.000 0086 from 0° to about 30° } See Recknagel's Table, page 126.
and at .000 0092 from 0° to 100° .

The correction amounting to only 0.92 mm. per metre per 100 degrees, may as a rule be neglected.

Correction for the Temperature of the Mercury. If our mercury-column measures P mm. at t° , it exerts the same pressure as a column of mercury of 0° , P_0 mm. high, where

$$P_0 = \frac{P}{(1 + kt)}$$

k stands for the mean coefficient of expansion of mercury from 0° to t° . According to Levi, k, for $t = 30$, is .000 181 43.

TABLE II. THE LOGARITHMS OF $(1 + .000 181 43 t)$, UP TO 30° .

t	Log. (1+kt)	t	Log. (1+kt)	t	Log. (1+kt)	P. P.	
						Δt	$\Delta \log.$
0°	.000 000	10°	.000 787	20°	.001 573		
1	079	11	866	21	652	.1	vi.
2	158	12	945	22	730	.2	8
3	236	13	.001 023	23	809	.3	16
4	315	14	102	24	887	.4	24
							31
5	394	15	180	25	965	.5	39
6	473	16	259	26	.002 044	.6	47
7	551	17	338	27	122	.7	55
8	630	18	416	28	201	.8	63
9	709	19	495	29	279	.9	71
10	787	20	573	30	357	1.0	79

TABLE III. VALUES OF k , $1 + kt$ AND $\text{LOG. } (1 + kt)$ FOR A FEW HIGHER TEMPERATURES, ACCORDING TO LEVI.

t	10 000 k.	1 + kt.	Log. (1 + kt).	Diff. Logs.	P.P. for Logs.	
					Δt	$\Delta \log.$
30°	1.8143	1.005 443	.002 357		1°	78
40	1.8149	1.007 260	.003 142	785	2	157
50	1.8157	1.009 078	.003 925	783	3	235
60	1.8165	1.010 900	.004 708	783	4	313
					5	391
70	1.8174	1.012 722	.005 490	782	6	470
80	1.8184	1.014 547	.006 272	782	7	548
90	1.8195	1.016 376	.007 054	782	8	626
100	1.8207	1.018 207	.007 836	782	6	704
					10	783
150	1.8281	1.027 421	—	—		
200	1.8378	1.036 755	—	—		

(3) *Correction for the Gravity at the Place of Observation.* Taking the gravity at lat. 45° and sea-level as unity, that for latitude ϕ , and the altitude of H metres above sea-level is*

$$G = (1 - .002\ 5935 \cos. 2\phi) (1 - .000\ 000\ 3142\ H) = 1 + \gamma.$$

Taking a and b as the computed values of the second term in the first and second factor respectively, and neglecting ab against a and b , we have, if

$$\phi \text{ is less than } 45^\circ; \quad G = 1 - a - b.$$

$$\phi \text{ is greater than } 45^\circ; \quad G = 1 + a - b.$$

By means of these formulæ we have calculated the following Table:—

* W. Lasch, Pogg. Annalen, Ergänzungsband III., p. 321, year 1853.

TABLE IV. RELATIVE VALUE OF GRAVITY AT SOME STATIONS.

STATION.	ϕ in Degrees and Decimals of 1°.	H Metres.	VALUE OF GRAVITY, G.	
			G at 45° and Sea-level = 1.	G in Paris = 1.
			$1 + \gamma$	$1 + \delta$
(0) Lat. 45°, and H = 0	45°	0	1.000 000	0.999 673
(1) Equator,	0	0	0.997 406	0.997 079
(2) Rome,	41.899°	29	.999 711	0.999 384
(3) Milan,	45.467	130	1.000 001	0.999 674
(4) Paris,*	48.837	60	1.000 327	1.000 000
(5) London,	51.519	50	1.000 569	1.000 242
(6) Berlin,	52.505	40	1.000 659	1.000 332
(7) Edinburgh, Princes Street, }	55.955	66	1.000 947	1.000 620
(8) Edinburgh, Calton Hill Observatory, }	55.955	107	1.000 934	1.000 607
(9) Glasgow, Exchange,	55.859	11	1.000 956	1.000 629
(10) Do. Observatory,	55.859	55	1.000 943	1.000 616
(11) Aberdeen, Univer- sity Laboratory, }	57.149	16	1.001 062	1.000 735
(12) Reikiavik,§	64.2	0	1.001 611	1.001 284
(13) Pole,	90°	0	1.002 594	1.002 267
"Edinburgh-Glasgow",	—	—	1.000 945	1.000 618

* Regnault's Laboratory.

§ ϕ taken from a map; H = 0 assumed.

|| Mean of the four values for G given under (7), (8), (9), and (10).

Examples to explain the use of the Table. (1) Regnault found that one litre of oxygen, measured off in his laboratory at 0° and under a pressure of 760 mm. of 0° , weighs 1.429 803 grammes. *This quantum* of oxygen will have the same weight anywhere else, because "weight," in chemistry, always means "mass"; but the pressure exerted by 760 mm. in Paris amounts to $760 \times 1.000\ 327$ units, hence one litre of oxygen of 0° , measured off under 760 mm. = 760 units at lat. 45 and sea-level, weighs only $1.429\ 803 \div 1.000\ 327 = 1.429\ 335$ grammes, and at any of our stations the weight becomes

$$1.429\ 803 \times \left\{ \frac{1 + \gamma}{1.000\ 327} = (1 + \gamma - .000\ 327) = 1 + \delta \right\} \text{ grammes.}$$

In Glasgow (No. 9), $\delta = 0.000\ 629$; hence the weight becomes 1.430 702.

(2) If 100°C be defined as the temperature at which the maximum tension of steam is equal to 760 units of pressure, the boiling point of water in Glasgow, under an atmospheric pressure of 760 mm. of 0° lies above 100° , because the water there boils under a pressure of $760 \times 1.000\ 956$ units. If the water is to boil at 100° exactly the barometer must stand at x mm., where x is determined by eq. $x \times 1.000\ 956 = 760$ units, whence $x = 759.27$.

In a gas-analysis the several gas-quanta are measured either in the state of dryness, or in a state of complete saturation with water. In the latter case, the observed gas-pressure P must be corrected by deducting from it the maximum tension of steam at the respective temperature, to obtain the tension which the *dry* gas would exhibit *at the same volume*. The first of the following set of three Tables gives the maximum tensions of steam at all temperatures that ever occur in gas-analysis, in absolute units. Hence the gas pressure " P " before being corrected, ought, strictly speaking, to be reduced to absolute units, but a little reflection (based upon a knowledge of what gas-analysis can do and cannot do) shows that the reduction, in ordinary practice, may be omitted.

TABLE V. TENSION OF VAPOUR OF WATER AT $t^{\circ}\text{C.}$; FROM $t = 0^{\circ}$ TO $t = 30.9^{\circ}\text{C.}$, AS CALCULATED BY BROCH FROM REGNAULT'S EXPERIMENTS.*

Taken from Landolt and Börnstein's Tables.

In Millimetres of Mercury, red. to 0° , and 45° Lat. and Sea Level.										
t	$^{\circ}0$	$^{\circ}1$	$^{\circ}2$	$^{\circ}3$	$^{\circ}4$	$^{\circ}5$	$^{\circ}6$	$^{\circ}7$	$^{\circ}8$	$^{\circ}9$
0°	4.57	4.60	4.64	4.67	4.70	4.74	4.77	4.80	4.84	4.87
1	4.91	4.94	4.98	5.02	5.05	5.09	5.12	5.16	5.20	5.23
2	5.27	5.31	5.35	5.39	5.42	5.46	5.50	5.54	5.58	5.62
3	5.66	5.70	5.74	5.78	5.82	5.86	5.90	5.94	5.99	6.03
4	6.07	6.11	6.15	6.20	6.24	6.28	6.33	6.37	6.42	6.46
5	6.51	6.55	6.60	6.64	6.69	6.74	6.78	6.83	6.88	6.92
6	6.97	7.02	7.07	7.12	7.17	7.22	7.26	7.31	7.36	7.42
7	7.47	7.52	7.57	7.62	7.67	7.72	7.78	7.83	7.88	7.94
8	7.99	8.05	8.10	8.15	8.21	8.27	8.32	8.38	8.43	8.49
9	8.55	8.61	8.66	8.72	8.78	8.84	8.90	8.96	9.02	9.08
10	9.14	9.20	9.26	9.32	9.39	9.45	9.51	9.58	9.64	9.70
11	9.77	9.83	9.90	9.96	10.03	10.09	10.16	10.23	10.30	10.36
12	10.43	10.50	10.57	10.64	10.71	10.78	10.85	10.92	10.99	11.06
13	11.14	11.21	11.28	11.36	11.43	11.50	11.58	11.66	11.73	11.81
14	11.88	11.96	12.04	12.12	12.19	12.27	12.35	12.43	12.51	12.59
15	12.67	12.76	12.84	12.92	13.00	13.09	13.17	13.25	13.34	13.42
16	13.51	13.60	13.68	13.77	13.86	13.95	14.04	14.12	14.21	14.30
17	14.40	14.49	14.58	14.67	14.76	14.86	14.95	15.04	15.14	15.23
18	15.33	15.43	15.52	15.62	15.72	15.82	15.92	16.02	16.12	16.22
19	16.32	16.42	16.52	16.63	16.73	16.83	16.94	17.04	17.15	17.26
20	17.36	17.47	17.58	17.69	17.80	17.91	18.02	18.13	18.24	18.35
21	18.47	18.58	18.69	18.81	18.92	19.04	19.16	19.27	19.39	19.51
22	19.63	19.75	19.87	19.99	20.11	20.24	20.36	20.48	20.61	20.73
23	20.86	20.98	21.11	21.24	21.37	21.50	21.63	21.76	21.89	22.02
24	22.15	22.29	22.42	22.55	22.69	22.83	22.96	23.10	23.24	23.38
25	23.52	23.66	23.80	23.94	24.08	24.23	24.37	24.52	24.66	24.81
26	24.96	25.10	25.25	25.40	25.55	25.70	25.86	26.01	26.16	26.32
27	26.47	26.63	26.78	26.94	27.10	27.26	27.42	27.58	27.74	27.90
28	28.07	28.23	28.39	28.56	28.73	28.89	29.06	29.23	29.40	29.57
29	29.74	29.92	30.09	30.26	30.44	30.62	30.79	30.97	31.15	31.33
30	31.51	31.69	31.87	32.06	32.24	32.43	32.61	32.80	32.99	33.18

* See note to Table VII., p. 135.

TABLE Va. SUPPLEMENT GIVING THE TENSIONS FROM 98° TO 102°.
 (Same Authority and Source, and same unit of pressure as above.)

t	°.0	°.1	°.2	°.3	°.4	°.5	°.6	°.7	°.8	°.9
98°	7 07.13	09.69	12.27	14.85	17.44	20.04	22.65	25.27	27.89	30.52
99	33.16	35.81	38.46	41.13	43.80	46.48	49.17	51.86	54.57	57.28
100	60.00	62.73	65.47	68.21	70.97	73.73	76.50	79.28	82.07	84.86
101	87.67	90.50	93.30	96.13	99.00	101.81	104.59	107.48	110.32	113.15
102	8 15.99									

To pass from unit of pressure to millimetres of mercury of 0° at a given place where $G = 1 + \gamma$, multiply the P by $(1 - \gamma)$.

TABLE VI. BOILING POINTS OF WATER AT PRESSURES P FROM 720 TO 800 MILLIMETRES OF MERCURY AT LAT. 45° AND SEA-LEVEL.

Centimetres.	MILLIMETRES.										
	0	1	2	3	4	5	6	7	8	9	
72	98°.50	.54	.58	.61	.65	.69	.73	.77	.80	.84	
73	.88	.92	.96	.99	1.03	1.07	1.11	1.14	1.18	1.22	
74	99.26	.30	.33	.37	.41	.44	.48	.52	.56	.59	
75	.63	.67	.71	.74	.78	.82	.85	.89	.93	.96	
76	100.00	.04	.07	.11	.15	.18	.22	.26	.29	.33	
77	.36	.40	.44	.47	.51	.55	.58	.62	.65	.69	
78	.73	.76	.80	.83	.87	.90	.94	.98	1.01	1.05	
79	101.08	.12	.15	.19	.22	.26	.30	.33	.37	.40	
80	.44										

In using this last Table for determining the 100°C point of a thermometer, do not forget to reduce the height of the barometer to 0°C. If the instrument is one of a high order, multiply the height of the barometer, as reduced to 0°, with the $(1 + \gamma)$ of the place of observation, to obtain the correct argument to enter the table with.

TABLE VII. MAXIMUM TENSION OF VAPOUR OF WATER,
FROM + 30° TO 100°.

(Same Authority and unit as before.)*

t	Tension.	Diff.	t	Tension.	Diff.	t	Tension.	Diff.
30°	31.51		55°	117.52	5.55	80°	354.87	14.14
31	33.37	1.86	56	123.29	5.77	81	369.51	14.64
32	35.32	1.95	57	129.31	6.02	82	384.64	15.13
33	37.37	2.05	58	135.58	6.27	83	400.29	15.65
34	39.52	2.15	59	142.10	6.52	84	416.47	16.18
35	41.78	2.26	60	148.88	6.78	85	433.19	16.72
36	44.16	2.38	61	155.95	7.07	86	450.47	17.28
37	46.65	2.49	62	163.29	7.34	87	468.32	17.85
38	49.26	2.61	63	170.92	7.63	88	486.76	18.44
39	52.00	2.74	64	178.86	7.94	89	505.81	19.05
40	54.87	2.87	65	187.10	8.24	90	525.47	19.66
41	57.87	3.00	66	195.67	8.57	91	545.77	20.30
42	61.02	3.15	67	204.56	8.89	92	566.71	20.94
43	64.31	3.29	68	213.79	9.23	93	588.33	21.62
44	67.76	3.45	69	223.37	9.58	94	610.64	22.31
45	71.36	3.60	70	233.31	9.94	95	633.66	23.02
46	75.13	3.77	71	243.62	10.31	96	657.40	23.74
47	79.07	3.94	72	254.30	10.68	97	681.88	24.48
48	83.19	4.12	73	265.38	11.08	98	707.13	25.25
49	87.49	4.30	74	276.87	11.49	99	733.16	26.03
50	91.98	4.49	75	288.76	11.89	100	760.00	26.84
51	96.66	4.68	76	301.09	12.33			
52	101.55	4.89	77	313.85	12.76			
53	106.65	5.10	78	327.05	13.20			
54	111.97	5.32	79	340.73	13.68			

* The original (we copy from Landolt and Börnstein) proceeds by tenth-degrees, and gives the pressures to four decimal places. At 50°, one hundredth of a mm. corresponds to .002 degrees, which surely exhausts all the precision of even Regnault's work.

TABLE VIII. BOILING POINTS t OF WATER UNDER DIFFERENT PRESSURES, P.

(From Zeuner, *Grundzüge der mechanischen Wärmetheorie*, 1877.)

"1 Atm." = Pressure of 760 mm. of mercury of 0° in Paris.

P	t	P	t	P	t
1 Atm.	100°	6 Atm.	159.2°	11 Atm.	184.5°
2 "	120.6	7 "	165.3	12 "	188.4
3 "	133.9	8 "	170.8	13 "	192.1
4 "	144.0	9 "	175.8	14 "	195.5
5 "	152.2	10 "	180.3		

TABLE IX. GIVING THE TENSION OF ABSOLUTE ALCOHOL FROM
 $t = 0^{\circ}$ TO $t = 30^{\circ}$

*Calculated after Regnault's determinations by L. Carius. Bunsen's
 Gasometrische Methoden.*

t	Tensions.	Diff.	t	Tensions.	Diff.
	Mm.*			Mm.	
0°	12.73		15°	32.60	1.91
1	13.65	.92	16	34.62	2.02
2	14.60	.95	17	36.77	2.15
3	15.59	.99	18	39.05	2.28
4	16.62	1.03	19	41.45	2.40
5	17.70	1.08	20	44.00	2.55
6	18.84	1.14	21	46.69	2.69
7	20.04	1.20	22	49.54	2.85
8	21.31	1.27	23	52.54	3.00
9	22.66	1.35	24	55.70	3.16
10	24.08	1.42	25	59.03	3.33
11	25.59	1.51	26	62.53	3.50
12	27.19	1.60	27	66.22	3.69
13	28.89	1.70	28	70.02	3.80
14	30.69	1.80	29	74.09	4.07
			30	78.41	4.32

* Millimetres of mercury of 0°C in Paris.

The original proceeds by tenth-degrees; but a glance at the numbers shows that the values between t and $t + 1$ were obtained by rectilinear interpolation.

For the gases enumerated in the following Table X. the exact relations between the weight D of a litre in grammes on the one hand, and the temperature t , the pressure P in millimetres of mercury of 0° and the gravity at the place of observation on the other, may be formulated thus; taking (D_0 Regnault) as a symbol for the D for $t = 0^{\circ}$ and $P = 760$ mm. of 0° in Regnault's laboratory,

$$D = (D_0 \text{ Regnault}) \times \frac{A}{A + t} \times \frac{P}{760} (1 + \delta); \text{ or}$$

$$D = \text{Const.} \frac{P}{A + t} (1 + \delta)$$

By means of this formula Table XI. has been calculated. In this Table the "logarithms" given are the logarithms of the constants.

TABLE X. PHYSICAL CONSTANTS OF A NUMBER OF GASES.

All, except the bracketed numbers, according to Regnault.

NAME AND FORMULA.	Coefficient of Expansion α at constant Pressure of about 760 mm.		Specific Gravity, determined at 0° and near 760 mm.			Weight of 1 Litre in Grammes (or Specific Gravity referred to that of Water of 4° as = 1000), if $t = 0^\circ$ and $P = 760$ mm. of Mercury of 0°.			
	100 α	$A = \frac{1}{\alpha}$	Air = 1	Oxygen = 16 Hydrogenium = 1	Hydrogen = 1	Lat. 45° and Sea-Level.	Regnault's Laboratory.	Berlin.	Glasgow and Edinburgh (G = 1.000 945)
Air, free of CO ₂ and H ₂ O	-	.367 06	1.000 000	14.471 4	14.437	1.292 78	1.293 204	1.293 63	1.294 00
Oxygen, O ₂	-	(.367 43)*	1.105 628	16.000 0	15.962	1.429 33	1.429 803	1.430 28	1.430 69
Nitrogen, N ₂	-	(.366 82)**	0.971 360	14.057 0	14.023	1.255 76	1.256 167	1.256 58	1.256 94
Hydrogen, H ₂	-	.366 13	0.069 268	1.002 4	1.000 0	0.089 55	0.089 578	0.089 61	0.089 63
Hydrogenium, H ₂	-		0.069 102	1.000 0	0.997 6	0.089 334	0.089 363	0.089 392	0.089 418
Carbonic Oxide, CO	-	.366 88	(0.967 79)§	(14.005 3)	(13.972)	(1.251 14)	(1.251 55)	(1.251 97)	(1.252 32)
Carbonic Acid	-	.370 99	1.529 084	22.128 0	22.075	1.976 77	1.977 418	1.978 07	1.978 64
Reference Mark,	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.

I. *At constant volume, Jolly; **At constant volume, Regnault. In I. and II. the fifth figure may be dropped without loss of real precision.

III. Lasch's re-calculation of Regnault's determinations, Pogg. Annal., Ergänzungsband III., p. 321; year 1853. § Baron Wrede, Berzelius' Jahresber., vol. 22, p. 72 (year of publication 1842). All other numbers concerning CO are calculated from this; except α which is Regnault's.

IX. See Table of Values G, p. 131. III. and VI. to IX. Presumable uncertainty about $\pm .000$ 1 in all cases.

I. to IX. Hydrogenium; an imaginary gas whose specific gravity and whose molecular weight is *exactly* equal to one-sixteenth of that of Oxygen.

TABLE XI. WEIGHT IN GRAMMES OF ONE LITRE OF GAS IF MEASURED
AT t° AND A PRESSURE OF P MM. OF MERCURY OF 0° .

According to Regnault's direct determinations.

Name of Gas.	Regnault's Laboratory.	Edinburgh-Glasgow.
Air free of CO_2 and H_2O .	$.463\ 579 \frac{P}{272.44 + t}$ Log. = .666 124 - 1	$.463\ 865 \frac{P}{272.44 + t}$ Log. = .666 392 - 1
Oxygen.	$.512\ 019 \frac{P}{272.16 + t}$ Log. = .709 286 - 1	$.512\ 334 \frac{P}{272.16 + t}$ Log. = .709 554 - 1
Nitrogen.	$.450\ 583 \frac{P}{272.61 + t}$ Log. = .653 775 - 1	$.450\ 861 \frac{P}{272.16 + t}$ Log. = .654 043
Hydrogen.	$.032\ 192\ 5 \frac{P}{273.13 + t}$ Log. = .507 755 - 2	$.032\ 212\ 4 \frac{P}{273.13 + t}$ Log. = .508 023 - 2
Carbonic Oxide.*	$.448\ 861 \frac{P}{272.57 + t}$ Log. = .652 112 - 1	$.449\ 138 \frac{P}{272.57 + t}$ Log. = .652 380
Carbonic Acid.	$.701\ 331 \frac{P}{269.55 + t}$ Log. = .845 923 - 1	$.701\ 763 \frac{P}{269.55 + t}$ Log. = .846 191 - 1

* Calculated after Baron Wrede's value of the specific gravity, Air = 1.

At temperatures from somewhere below 0°C to 100°C , and pressures not exceeding one atmosphere, the formulæ given in this Table afford a closer approximation to the truth, than the general equation for "*perfect gases*" given as No. VII. on page 140. At temperatures *above* 100° , it is probably the reverse, in general at least.

RULES CONCERNING "PERFECT" GASES.

No gas is a "perfect gas" in an absolute sense, but any gas, if at a moderate pressure P its temperature be raised sufficiently above its point of condensation at P , becomes "*perfectly gaseous*," which, in practice, means that within a certain area of combinations of temperature and pressure, which includes at least all pressures from P downwards and all temperatures from t° upwards, it obeys the law ;

$$\frac{VP}{T} = Q ; \text{ or } V = \frac{T}{P} Q ; \text{ or } P = Q \frac{T}{V} \quad \text{I.}$$

V stands for the volume at the pressure P and the "absolute temperature," *i.e.*, the temperature counted from the absolute zero, which, in practice is $T = 273 + t$. The ratio $\frac{T}{P}$ is called the "*disgregation*" of the gas,

Q is a constant which may be defined, *arithmetically*, as the volume of the gas at $T = 1^\circ$ and $P = 1$ mm., or as the pressure of the gas at $V = 1$ and $T = 1$; or *physically*, as the volume which the gas occupies whenever the disgregation $T : P = 1$, it being understood that the combination T, P falls within the area of perfect gaseousness ; or as the pressure which the gas exhibits whenever $T = V$, subject to the qualification named. The equation, in the case of a mixed gas, holds for any one of the components, in the sense of the following explanation :—

(*Mixed Gases.*) Given a mixture of the (unitary or themselves mixed) gases I., II., III., ... N, and supposing, at a given constant temperature t° , $v_1, v_2, v_3, v_4, \dots$ are the partial volumes of I., II., etc., *i.e.*, the volumes which they would occupy at t° and P mm. if they were floating upon one another in layers, while $p_1, p_2, p_3, \dots$ are the partial pressures, meaning the pressures which the components severally would exert, if each of them filled the V units of space by itself (at t°), we have, as well established propositions,

$$v_1 + v_2 + v_3 + \dots = V \quad \text{II.}$$

$$p_1 + p_2 + p_3 + \dots = P \quad \text{III.,}$$

and for any one of the components,

$$\frac{V}{v} = \frac{P}{p} \quad \text{IV.,}$$

$$\text{whence } v_1 : v_2 : v_3 : \dots = p_1 : p_2 : p_3 : \dots \quad \text{IVa.}$$

If we define the "*specific gravity*" of a gas-species as meaning the weight of a given volume, measured by the weight of the same volume of some standard gas of the same disgregation, then by eq. I., the specific gravity S is independent of P and t . As shown by *Avogadro*, S is a function of only the molecular weight M thus :

$$S' : S'' : S''' : \dots = M' : M'' : M''' : \dots \quad \text{and consequently,}$$

$$S' = k M' ; S'' = k M'' ; S''' = k M''' , \dots \quad V.$$

where k is a constant which depends only on the units chosen for S and M .

Relation between Weight and Volume. According to Regnault, one litre of oxygen, if measured off in Paris at 0° and under a pressure of 760 mm. of mercury of 0° , weights 1.429 803 grammes. (Log. = .155 276.) Hence at the pressure of P mm. and the absolute temperature T we have for the weight of one litre of a gas of the molecular weight M the value

$$D = \frac{P}{T} \times \frac{1.429\ 803 \times 273}{760} \times \frac{M}{O_2} \quad \text{VI.}$$

The corresponding value for any other place than Paris is found by multiplying the expressions by $(1 + \delta)$. (See Table IV., p. 131.) Hence taking the values M in reference to $O = 16$, and contracting constants, we have generally

$$D = \frac{P}{T} (1 + \delta) \times 0.032\ 100 \times \frac{1}{2} M \text{ grammes,} \quad \text{VII.}$$

and for the *volume of M grammes*

$$\frac{T}{P} \times 62.305\ 3 (1 - \delta) \text{ litres,} \quad \text{VIII.}$$

TABLE XII. VALUES OF THE CONSTANTS IN EQS. VII. AND VIII. FOR A NUMBER OF STATIONS.

Station.	$\delta = .000$	Const. VII.	Log. Const. VII.	Const. VIII.	Log. Const. VIII.
Lat. 45° and Sea-Level, -	IV.	*		*	
Paris, 000	- 327	.032 089 5	.506 363 - 2	62.325 7	1.794 667
London, + 242	.000	.032 100 0	.506 505 - 2	62.305 3	1.794 525
Berlin, - + 332	.032 107 8	.506 610 - 2	62.290 2	1.794 419	
Edinburgh- }	+ 332	.032 110 7	.506 650 - 2	62.284 6	1.794 380
Glasgow, }	+ 618	.032 119 8	.506 773 - 2	62.266 8	1.794 257

* For ordinary work the last two figures may be cut off.

By means of const. VII. we can easily calculate the value D for any mixture of gases of known molecular weights if the volumetric composition of the mixture be given; in the case of air, however, it is better to go back to Regnault's direct determination for $t = 0^\circ$ and $P = 760$ mm. We then arrive at the formula

$$(\text{for Air}) D = \frac{P}{T} \times 0.464\ 532^* \times (1 + \delta) \quad \text{IX.}$$

$$\text{Log.} = .667\ 016 - 1$$

For Edinburgh-Glasgow, we have

$$D = \frac{P}{T} \times .464\ 819 \quad \text{X.}$$

$$\text{Log.} = .667\ 284 - 1$$

* See foot-note on page 141.

From VIII. the volume of one gramme of Air is

$$\frac{T}{P} \times 2.15270 \times (1 - \delta) \text{ litres,} \quad \text{XI.}$$

For $T = 273^\circ$ and $P = 760 \text{ mm.}$, we have

Volume of 1 gramme of Air = .773 273 litres in Paris, and
 $.773\,273 \times (1 - \delta)$ anywhere else. $\text{Log.} = .888\,333 - 1$

The factors in the following Table are calculated from Regnault's values for the weight of one litre of oxygen at 0° and 760 mm. in Paris, and the specific gravity of oxygen (Air = 1) and the values H_2 and O_2 , as determined chemically. ($H_2 = 2.004\,8$; $O_2 = 32.$) "M" always stands for the value of the molecular weight if $O = 16$.

* The disagreement between the constant in eq. IX. and the corresponding constant given in Table XI., page 138, is of less significance than it appears at first sight. Both formulæ give the same (viz. Regnault's) value for the weight of one litre of air at 0° and 760 mm. , and consequently must give very nearly the same value at any temperature and pressure not far removed from 0° and 760 .

TABLE XIII. FOR THE REDUCTION OF GASOMETRIC CONSTANTS TO ONE ANOTHER.

A.—Given: The Molecular Weight, M ; $O = 16$.

Wanted.	Formula.
(1) Specific Gravity, S_0 ; Oxygen = 16	$S_0 = 0.5 M.$
(2) Specific Gravity, S_H ; Hydrogen = 1	$S_H = 0.49\ 880 M.$ $\text{Log.} = .697\ 93 - 1$
(3) Specific Gravity, S_A ; Air = 1	$S_A = 0.034\ 551 M.$ $\text{Log.} = .538\ 46 - 2$
(4) Weight of 1 Litre in Grammes, D_0 if $t = 0$, and $P = 760$ mm. (0°)	$D_0 = 0.044\ 681 M \times (1 + \delta)$ $\text{Log.} = .650\ 13 - 2$
(4a) Value D'_0 in Edinburgh-Glasgow.	$D'_0 = 0.044\ 709$ $\text{Log.} = .650\ 39 - 2$
(5) Weight W of 1 Litre, in Grammes, at $T:P = 1$	$W = 0.016\ 050 M \times (1 + \delta)$ $\text{Log.} = .205\ 475 - 2$
(5a) Value W' for Edinburgh-Glasgow.	$W' = 0.016\ 060 M.$ $\text{Log.} = .205\ 74 - 2$
(6) Weight D_{15} of 1 Litre at $t = 15^\circ$ and $P = 760$ mm.	$D_{15} = 0.042\ 354 M \times (1 + \delta)$ $\text{Log.} = .626\ 90 - 2$
(6a) Value D'_{15} for Edinburgh-Glasgow.	$D'_{15} = 0.042\ 380 M.$ $\text{Log.} = .627\ 17 - 2$

B.—*Wanted: The Molecular Weight, M ; $O = 16$.*

Given.	Formula.
(1) Specific Gravity, S_0 ; Oxygen = 16	$M = 2 S_0$
(2) Specific Gravity, S_H ; Hydrogen = 1	$M = 2.0048 S_H$; Log. .302 07
(3) Specific Gravity, S_A ; Air = 1	$M = 28.943 S_A$; Log. = 1.461 54
(4) Weight of 1 Litre in Grammes, D_0 of $t = 0^\circ$ and $P = 760$ mm.	$M = 22.381 D_0 (1 - \delta)$ Log. = 1.349 87
(4a) Value D'_0 for Edinburgh-Glasgow.	$M = 22.367 D'_0$ Log. = 1.349 61
(5) Weight W of 1 Litre in Grammes at $T : P = 1$	$M = 62.305 W (1 - \delta)$ Log. = 1.794 525
(5a) Value W' for Edinburgh-Glasgow.	$M = 62.267 W'$ Log. = 1.794 26
(6) Weight D_{15} of 1 Litre at $t = 15^\circ$ and $P = 760$ mm.	$M = 23.6104 D_{15} (1 - \delta)$ Log. = 1.373 10
(6a) Value D'_{15} for Edinburgh-Glasgow.	$M = 23.596 D'_{15}$ Log. = 1.372 83

C.—Given: The Weight of One Litre W at $T:P = 1$.

Wanted.	Formula.
(1) Weight of 1 Litre D_0 at 0° and 760 mm.	$D_0 = 2.783\ 9\ W$ Log. = .444 65
(2) Weight of 1 Litre D_{15} at 15° and 760 mm.	$D_{15} = 2.638\ 9\ W$ Log. = .421 42
(3) Specific Gravity, S_0 ; Oxygen = 16	$S_0 = 31.152\ 6\ W\ (1 - \delta)$ Log. = 1.493 50
(4) Specific Gravity, S_A ; Air = 1	$S_A = 2.152\ 7\ W\ (1 - \delta)$ Log. = .332 98

D.—Change of Standard for Specific Gravity S .

Given S in reference to	Wanted S in reference to	Multiply by f	
		f	Log. f
Oxygen = 16 Oxygen = 16	Hydrogen = 1 Air = 1	.997 61 .069 102	.998 96 - 1 .839 49 - 2
Air = 1 Air = 1	Oxygen = 16 Hydrogen = 1	14.471 4 14.436 8	1.160 51 1.159 47
Hydrogen = 1 Hydrogen = 1	Oxygen = 16 Air = 1	1.002 4 .069 267	.001 04 .840 53 - 2

GAS-ANALYSIS.

(I)—Measurement of Gas-Quanta.

To "measure" a gas we bring it under conditions which bring it sufficiently near the state of perfect gaseousness, and determine its volume V , temperature $T = 273 + t$, and pressure P . We then reduce the volume to some standard disgregation to find what we will call the "perfect-gas quantity," or, shortly, the "quantity" of the gas. If $T_0 : P_0$

is the chosen disgregation and V_* the reduced volume, the formula for the reduction is

$$V_* = \frac{V P}{T} \times \frac{T_0}{P_0} = Q \times \frac{T_0}{P_0} \quad (\text{See eq. I., page 139}).$$

The quantity may be read as a volume or as a pressure, because Q is the pressure at unit volume as well as the volume at unit pressure (and $T = 1$ on both sides). Hence, supposing we reduce a given gas to T_0 degrees and (say) 300 mm., the reduced volume is

$$V_* = Q \frac{T_0}{300}$$

Now, for the same gas, the pressure at T_0 degrees and the volume 300 is $= P_*$ and $P_* = Q \frac{T_0}{300} = V_*$, numerically. But in any case the

factor $\frac{T_0}{P_0}$ is a mere encumbrance; we had better simply calculate

$Q \left(= \frac{V P}{T} \right)$ and call it the "*quantity*" or "*reduced volume*." If

desirable to have the volume at "0° and 760 mm.," multiply Q by $\frac{273}{760} = .35921$; $\text{Log.} = .555349 - 1$. If, as usual, the gas to be

measured is saturated with vapour of water, the observed P must be corrected by deducting the tension of steam τ at the respective temperature, so that we have $Q = \frac{V(P - \tau)}{T}$.

A particular case presents itself if the quantity of water produced in the combustion of a gas has to be determined. We then measure the product of combustion in two ways, namely, (1) in the cold; let the numbers be $V' P' T'$. By computing the value $\frac{V'(P' - \tau)}{T'} = V'_0$, we

find the quantity of the dry part of the gas. We then (2) repeat the measurement at a high enough temperature and a low enough pressure, to convert the water into a practically perfectly gas, and from the numbers (let them be $V'' T'' P''$) calculate the quantity of the whole, by computing $V''_0 = \frac{V'' P''}{T''}$. The quantity of the steam is $= V''_0 - V'_0$.

In the second measurement, however, the observed values for volume and pressure have to be corrected. The former, because the capacity (V) of the eudiometer down to the respective mark r at t'' ($= T'' - 273$) is greater than that recorded in the calibration table as V for the lower temperature t . We obviously have $(V) = V \left\{ 1 + \gamma (t'' - t) \right\}$ where γ stands for the coefficient of cubic expansion of the glass, which, for

$t'' = 100$ to 130° may be put down at .000 027 6. To avoid misplacement of the decimal point remember that the expansion is 2.76 units, per 100° increase of temperature, per 1000 volumes.

Correction of P.—Supposing the column of mercury suspended in the eudiometer measures h mm.-divisions of the scale, and the other measurements of pressure in connection with the combustion in hand were all made at the temperature t , or, in average, at t degrees (e.g., at 15°), the observed number h must be multiplied with the factor $\left\{1 + \lambda (t'' - t)\right\}$ to reduce them to the general unit of length adopted. $\lambda = \frac{1}{3}$ of $\gamma = .000\ 009\ 2$, i.e., the correction comes to 0.92 mm. per 1000 mm. per 100° . In ordinary practice this correction may be neglected. More important is the following:—

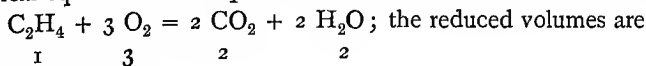
The general unit of pressure adopted for the measurements is that exerted by 1 mm. of mercury of t° , but our h mm. of mercury of t'' degrees do not exert h such units of pressure, but only (h) where (by formula on page 129)

$$(h) = \frac{h(1 + kt)}{1 + kt''} = h \left\{1 - k(t'' - t)\right\}$$

Supposing the barometer to stand at B mm. at t° , we have for P'' the value $B - (h)$, and as $V'' = (V)$, for the true quantity of the mixture of steam and gas $V_0'' = \frac{(V)[B - (h)]}{T''}$

Notes on the Method of Combustion.—Avogadro's law enables us to read the volumetric composition of a gas in its formula. Thus, for instance, a glance at the formula of methylamine $\text{CH}_5\text{N} = 31$, shows to us that if H_2 grammes* of hydrogen occupy one reduced volume, i.e., represent one gasometric unit of quantity, N_2 grammes of nitrogen do the same, etc., and that, consequently, one volume of methylamine contains 2.5 $\text{H}_2 = 2.5$ volumes of hydrogen, 0.5 $\text{N}_2 = 0.5$ volume of nitrogen, and 0.5 $\text{C}_2 =$ "half a volume of carbon-gas," meaning the quantity of carbon contained in one volume of carbonic acid, or marsh-gas, or one-carbon gas generally.

The volume of oxygen required for the combustion of one volume of a named gas-species, the contraction involved in the combustion, the volume of CO_2 produced, etc., are easily seen from the respective chemical equation. *Example:*



Hence contraction = $c = (1 + 3) - 2 = 2$

* It is hardly necessary to point out that, in the present connection, H_2 grammes of hydrogen mean only the weight of what, at the adopted standard disgregation, is one reduced volume or one unit of hydrogen; and similarly in similar cases.

Yet, in practical gas-analysis, the following Table will be found convenient. Part A. of the Table reads as follows:—Every 1 volume of the gas named in column I. requires for its combustion s_0 volume of oxygen; the contraction involved = c volume, the volume of steam produced is w, that of the CO_2 = k, that of the nitrogen = n. In the second part B. (which refers to gases combustibile by hydrogen) h stands for the volume of hydrogen required for the combustion of 1 volume of gas.

TABLE XIV. COMBUSTION-CONSTANTS OF A NUMBER OF GASES.

A.—Gases Combustible by Oxygen.

	c.	k.	s_0 .	w.	n.
Hydrogen, H_2 ,	1.5	0	0.5	1	0
Carbonic oxide, CO ,	0.5	1	0.5	0	0
Methyl-aldehyde, CH_2O ,	1	1	1	1	0
Ammonia, NH_3 ,	1.25	0	0.75	1.5	0.5
Methylamine, CH_5N ,	1.75	1	2.25	2.5	0.5
Cyanogen, N_2C_2 ,	0	2	2	0	1
Hydrocyanic acid, NCH ,	0.75	1	1.25	0.5	0.5
Marsh Gas, CH_4 ,	2	1	2	2	0
Acetylene, C_2H_2 ,	1.5	2	2.5	1	0
Ethylene, C_2H_4 ,	2	2	3	2	0
Ethane, C_2H_6 ,	2.5	2	3.5	3	0
Propylene, C_3H_6 ,	2.5	3	4.5	3	0
Propane, C_3H_8 ,	3	3	5	4	0
Oxide of methyl, $\text{C}_2\text{H}_6\text{O}$,	2	2	3	3	0
Benzol, C_6H_6 ,	2.5	6	7.5	3	0
Gas, $\text{Ca H}_\beta = 1$ vol.,	$1 + \frac{\beta}{4}$	α	$\alpha + \frac{\beta}{4}$	$\frac{\beta}{2}$	0
Gas, $\text{Ca H}_\beta \text{ O}\gamma = 1$ vol.,	$1 + \frac{\beta}{4} - \frac{\gamma}{2}$	α	$\alpha + \frac{\beta}{4} - \frac{\gamma}{2}$	$\frac{\beta}{2}$	0

B.—Gases Combustible by Hydrogen.

	c.	h.	w.	n.
Oxygen, O_2 ,	3	2	2	0
Ozone, O_3 ,	4	3	3	0
Nitrous oxide, N_2O ,	1	1	1	1
Nitric Oxide, NO ,*	1.5	1	1	0.5

* The entries for NO have a purely theoretical significance, because this gas does not explode with hydrogen.

The Method of Combustion is a method of ultimate analysis, which, in the case of gases of class A., consists in this, that, starting with a known quantity of the gas to be analysed, we add a known sufficient quantity of oxygen, fire the mixture with an electric spark, and, besides measuring the product of combustion, determine the carbonic acid, the nitrogen, and, if necessary, the total water present in it. Before explaining the arithmetic of the subject, let us adopt the formula $\alpha \text{ C}_2, \beta \text{ H}_2, \gamma \text{ O}_2, \delta \text{ N}_2 = 1$ volume as expressing the as yet unknown composition of the gas, by saying that every unit (reduced volume) of the gas contains α units of carbon, β units of hydrogen, γ units of oxygen, δ units of nitrogen-gas, potentially of course; the state of combination of the several elements we will suppose to be unknown. In this case we must determine the following gas-quantities:—

	Units or reduced volumes.
(0) That of the sample under analysis,	V
(1) That of the mixture of gas and oxygen,	V ₁

After firing.

(2) That of the product, measuring cold and deducting the tension of H ₂ O from P, as usual,	V ₂
(3) That of the total product, measuring at a sufficiently high temperature to convert the steam into a perfect gas,	V ₃

After removing the CO₂.

(4) That of the CO ₂ -free residue,	V ₄
--	----------------

The residue (4) which consists of nitrogen and oxygen, must be analysed, either by absorbing the oxygen with pyrogallate and measuring the nitrogen, or by firing the gas with a sufficiency of measured hydrogen, and determining the contraction; one-third of it is the quantity of the oxygen. Suppose

(5) The nitrogen found amounts to	V ₅
-----------------------------------	----------------

Taking K, $\mathfrak{H}$, W as representing the quantity of carbonic acid, nitrogen and steam produced, respectively, we easily see that,

$$W = V_3 - V_2; \text{ whence } \beta = \frac{1}{V} W, \quad (1)$$

$$\mathfrak{H} = V_5 \quad (2)$$

$$K = V_2 - V_4; \text{ and } \alpha = \frac{1}{V} \frac{K}{2} \quad (3)$$

The oxygen is calculated as follows:—Let S_0 denote that part of the added oxygen, which, conjointly with the oxygen of the substance, served to burn the carbon and the hydrogen, and S_r denote the surplus oxygen added, so that $S_0 + S_r = S =$ total oxygen added, then,

$$V_1 = V + S_0 + S_r$$

$$V_2 = \mathfrak{X} + K + S_r$$

$$V_1 - V_2 = C = V + S_0 - \mathfrak{X} - K; \quad (4)$$

$$\text{and, as } K = V_2 - V_4; \quad V_1 - V_4 = V + S_0 - \mathfrak{X} \quad (5)$$

Now it was obviously the oxygen-sum $S_0 + V\gamma$ which produced the CO_2 and the H_2O , hence,

$$S_0 + V\gamma = K + \frac{1}{2}W; \text{ or } \gamma = \frac{1}{V}(K + 0.5W - S_0) \quad (6)$$

C , K and $\mathfrak{X}$ being known, S_0 can be calculated from eq. (4) or eq. (5).

If nitrogen and oxygen be absent, we need determine only K and C , because eq. (4) becomes

$$C = V + S_0 - K; \text{ and, as } S_0 - K = \frac{1}{2}V\beta$$

$$\beta = \frac{2(C - V)}{V}$$

On the other hand, whenever oxygen is present W must be determined, unless such determination is rendered superfluous by independent determinations by other methods than the method of combustion. Supposing for instance the gas to be analysed is $\alpha \text{C}_2 \cdot \beta \text{H}_2 + \epsilon (\text{CO}) = 1$ volume, the determination of only C and K does not enable us to calculate the *three* unknown quantities α, β, ϵ ; we must add the determination of W , unless we know that there is CO , and by determining it by absorption with cuprous chloride, eliminate ϵ as an unknown quantity. Sometimes the determination of W in addition to C and K is the only method for determining the oxygen. Supposing for instance we even know that one of the components is ethylene, C_2H_4 , or oxide of methyl $\text{C}_2\text{H}_6 \text{ O} = \text{C}_2\text{H}_4 + \text{H}_2\text{O}$, there is no other way for determining the latter component.

In order now to show how far the method of combustion goes as an indirect method of *proximate quantitative analysis*, let us assume, in the first instance, we had to deal with a mixture of gases I., II., III., etc., all falling under the general formula $\alpha \text{C}_2 \cdot \beta \text{H}_2 \cdot \gamma \text{O}_2 \cdot \delta \text{N}_2 = 1$ molecule, and that the special formulæ of I., II., etc., were known to us. To find the per-unitages $x' x'' x'''$, etc., of the several components, we might begin by determining the coefficients $\alpha, \beta, \gamma, \delta$, for the given mixture, and then establish equations between these on the one hand and the special values

$\alpha' \alpha'' \dots \beta' \beta'' \dots \gamma' \gamma'' \dots$ for I., II., III., etc., on the other, thus :

$$\begin{aligned} \alpha &= \alpha'x' + \alpha''x'' + \alpha'''x''' \dots \&c., & \text{I.} \\ \beta &= \beta'x' + \beta''x'' + \beta'''x''' \dots \&c., & \text{II.} \\ \gamma &= \gamma'x' + \gamma''x'' + \gamma'''x''' \dots \&c., & \text{III.} \\ \delta &= \delta'x' + \delta''x'' + \delta'''x''' \dots \&c., & \text{IV.} \\ \mathbf{I} &= x' + x'' + x''' \dots \&c., & \text{V.} \end{aligned}$$

By means of these five equations *we may* be able to find the five values x for a mixture of five gases, but it does not follow that we always are. Supposing, for instance, the two elements N and O are absent, eqs. III. and IV. vanish and only three equations are left. And, if it happens that each of the dashed values β is say k times the corresponding α (k being a constant), or if, for all the components, there is a constant relation between α and β , such as $\beta = 2\alpha + 1$, eq. II. collapses, so that only two equations are left. A similar result occurs if all the components contain the same number of atoms of carbon or of hydrogen per molecule. Supposing, for instance, the gas is known to be a mixture of propylene, di-methyl and benzol-vapour, eq. II. becomes

$$3 = 3x' + 3x'' + 3x''', \text{ i.e., a mere repetition of eq. V.}$$

The general rule is, to first of all reason out, how many and which of the quantities $K, C, \mathfrak{J}, W$, in addition to our *knowledge of the constitution of the gas* we should need for finding the coefficients $\alpha, \beta, \gamma, \delta$, for the gas-mixture. If 4, 3, 2, suffice, then (in general) 3, 2, 1, of the eqs. I. to IV., together with V., will suffice to find the 4, 3, 2, unknown quantities. There is however no need of our calculating the empirical elementary composition of our mixture at all; we had better establish equation between the experimentally ascertained values $c, k, \mathfrak{n}, w = \frac{1}{V} (C, K, \mathfrak{J}, W)$ on the one hand, and the special values $c', c'' \dots k', k'' \dots \mathfrak{n}', \mathfrak{n}'' \dots w', w'' \dots$ on the other, thus :

$$\begin{aligned} k &= k'x' + k''x'' \dots \&c., & \text{Ia.} \\ c &= c'x' + c''x'' \dots \&c., & \text{IIa.} \\ w &= w'x' + w''x'' \dots \&c., & \text{IIIa.} \\ \mathfrak{n} &= \mathfrak{n}'x' + \mathfrak{n}''x'' \dots \&c., & \text{IVa.} \\ \mathbf{I} &= x' + x'' \dots \&c., & \text{Va.} \end{aligned}$$

and solve *these* equations in respect to x', x'' , etc.

GAS ABSORPTION.

Bunsen's Law may be explained by reference to the following fictitious experiment :—A glass vessel of $v + h$ volumes capacity is charged with a mechanical mixture of the gases I., II., III., etc.,

h volumes of a gas-free liquid are now introduced, so that the mixed gas is compressed into v volumes and its (dry) pressure raised to P mm. at t° , which temperature from now remains constant. Gas and liquid are now shaken together until absorptiometric equilibrium is established. The effect is, that the gas-space is saturated with the vapour of the liquid, while the (dry-gas) pressure in it is reduced (from P) to a less value p , because, of each of the gas-components I., II., III., etc., a portion q has passed into solution, and only the rest r remained in the gas-space. The general law of distribution is complex, but, if the solvent is water or alcohol, if none of the gases act chemically upon, or are abundantly soluble in, the liquid, if the pressure P is less than (say) 1 atmosphere and t lies between 0° and (about) 30° C, then according to Bunsen, the quantity of any one of the several gases I., II., III., etc., which is in solution, is governed by the equation

$$q = h \beta \pi \quad \text{I.}$$

where π stands for the partial pressure of this component in the gas-rest. β and q are of course of the same denomination: if q means milligrammes, β means milligrammes likewise, but we will agree upon measuring *all* gas-quanta by *volume* at (virtually) $t = 0^\circ$ and 1 mm. pressure, so that " q " units of gas means q times the quantity which fills unit of volume at $t = 0^\circ$ and $P = 1$ mm., or at the disgregation 273, and on the basis of this stipulation, with Bunsen, call β the "*coefficient of absorption*" of the gas at t° . β , in virtue of an as yet unknown law, increases, when the temperature falls, and is in general, greater for alcohol than for water.

Assuming now that every one volume of the original mixed gas contains m' volume of I., m'' of II., m''' of III., etc. (in the sense in which volume of air contains 0.21 volume of oxygen and 0.79 of nitrogen), it is clear beforehand, that the corresponding per-unitages in the gas-rest and those in the dissolved gas have different values. Let those for the *dissolved gas* be s' s'' s''' and those for the *gas-rest* n' n'' n'''

In discussing our equation let us begin with a limit-case.

I. v is so very much greater than h , that the process of absorption involves no appreciable change of composition in the gas-mixture. In this case the dissolved portions of the several individual gases are $q' = p h (m' \beta')$; $q'' = p h (m'' \beta'') + p h (m''' \beta''')$, etc.; the total quantity of dissolved gas is

$$Q = p h (m' \beta' + m'' \beta'' + m''' \beta''', \text{ etc.}) \quad \text{II.}$$

and the per-untage in it of any one of the components is

$$s = \frac{q}{Q} = \frac{m \beta}{\sum m \beta} \quad \text{III.}$$

II. We will now return to our original conception and assume $h : v$ to be finite. In this case, as quite generally $\pi = n p$, eq. I. reads

$$q = n p (h \beta) \quad \text{IV.}$$

and for the undissolved rest of the respective gas we have

$$r = v n p \times \frac{273}{273 + t} \text{ which we will abbreviate into}$$

$$r = v_0 n p \quad \text{V.}$$

$$\text{hence } q + r = n p (v_0 + \beta h) \quad \text{VI.}$$

But $q + r$ is a function of P and m thus :

$$q + r = v_0 m P$$

$$\text{hence } v_0 m P = n p (v_0 + \beta h) \quad \text{VII.}$$

$$\text{or } n = m \frac{P v_0}{p (v_0 + \beta h)} \quad \text{VIIa.}$$

$$\text{or } \frac{P}{p} = \frac{n (v_0 + \beta h)}{m v_0} \quad \text{VIIb.}$$

By summing up the series of special equations of which VIIa. is the general form, we have

$$n' + n'' + n''' \dots = 1 = \frac{P}{p} \left(\frac{m'}{1 + \frac{\beta h}{v_0}} + \frac{m''}{1 + \frac{\beta h}{v_0}} \dots \right) \quad \text{VIII.}$$

$$\text{whence } \frac{P}{p} = 1 \div \sum \frac{m}{1 + \frac{\beta h}{v_0}} \quad \text{IX.}$$

By substituting this for the $\frac{P}{p}$ in eq. VIIa. we have

$$n = \frac{m}{1 + \frac{\beta h}{v_0}} \frac{1}{\Sigma} \quad \text{X.}$$

where ' Σ ' stands for the sum which figures in eq. IX.

From VIIa. we see that $\frac{n}{m}$ depends only on the ratio $\frac{P}{p}$ but is independent of the absolute values of P and p . The total quantity of gas dissolved is $Q = P v_0 - p v_0$, whence

$$Q = v_0 P (1 - \Sigma), \quad \text{XI.}$$

$Q \div h p$ is what, in the case of a unitary gas, would be the coefficient of absorption; in the case of mixed gas, however, the coefficient—we will call it λ —is variable, being

$$\lambda = \frac{Q}{h p} = \frac{1 - \Sigma}{\frac{h}{v_0} \Sigma} \quad \text{XIa.}$$

By equations IV. and VIIa. we have, for any one of the components,

$$q = h \beta \left(\frac{m P v_0}{v_0 + \beta h} \right) \quad \text{XII.}$$

$$\text{hence } s = \frac{q}{Q} = \frac{h \beta m}{(v_0 + \beta h)(1 - \Sigma)} \quad \text{XIII.}$$

COMPOSITION OF AIR AND ABSORPTION OF ITS COMPONENTS BY WATER.

For laboratory purposes the mean composition of *dry air* may be assumed to be

	By Volume.	By Weight.
Oxygen,	20.90	23.14
Nitrogen,	79.10	76.86
	100.00	100.00
Carbonic acid,	0.03	0.046
	100.03	100.046

If one volume of water of t° is shaken with constantly renewed portions of a mixture containing m_1 of oxygen, m_2 of nitrogen, and m_3 of carbonic acid per unit volume until absorptiometric equilibrium is established and the dry-gas pressure at the end is p mm., the water dissolves of

Oxygen,	$m_1 p \beta_1$ volumes.
Nitrogen,	$m_2 p \beta_2$ „
Carbonic acid,	$m_3 p \beta_3$ „

measured at 0° and 1 mm. dry pressure, the β 's being the "coefficients of absorption." The three absorbed gas volumes, if measured at p mm., occupy $m_1 \beta_1$, $m_2 \beta_2$, $m_3 \beta_3$ volumes. Hence, at any pressure, unit volume of the *absorbed gas* contains :

$$\begin{aligned} \text{Oxygen, } n_1 &= \frac{m_1 \beta_1}{m_1 \beta_1 + m_2 \beta_2 + m_3 \beta_3} \\ \text{Nitrogen, } n_2 &= \frac{m_2 \beta_2}{m_1 \beta_1 + m_2 \beta_2 + m_3 \beta_3} \\ \text{Carbonic acid, } n_3 &= \frac{m_3 \beta_3}{m_1 \beta_1 + m_2 \beta_2 + m_3 \beta_3} \end{aligned}$$

The following coefficients of absorption were determined, those of nitrogen and oxygen by Dittmar* (absorption experiments with *air* at 1 atm. pressure), that of carbonic acid by Bunsen (by experiments with the pure gas at varying pressures from 1 atm. downwards):—

* Challenger Memoirs, vol. of "Physics and Chemistry," pp. 172 and 173.

TABLE XV. COEFFICIENTS OF ABSORPTION BY WATER OF

Oxygen.		Nitrogen.		Carbonic Acid.		Air, free from CO ₂ .			
Symbols,	β_1		β_2		β_3		λ		
t	1000 β_1	Diff.	1000 β_2	Diff.	1000 β_3	Diff.	1000 λ	Diff.	100 n ₁ *
0°	49.03		24.40		1797		29.54		34.69
1	47.70	-1.33	23.78	-.62	1721	-76	28.78	-.76	.65
2	46.45	1.25	23.21	.57	1648	73	28.06	.72	.60
3	45.25	1.20	22.65	.56	1579	69	27.37	.69	.55
4	44.11	1.14	22.12	.53	1513	66	26.72	.65	.51
5	43.03	1.08	21.62	.50	1450	63	26.09	.63	.47
6	41.99	1.04	21.14	.48	1390	60	25.50	.59	.42
7	41.00	.99	20.69	.45	1334	56	24.93	.57	.38
8	40.06	.94	20.25	.44	1281	53	24.39	.56	.33
9	39.15	.91	19.83	.42	1231	50	23.87	.52	.28
10	38.28	.87	19.43	.40	1185	46	23.37	.50	.24
11	37.45	.83	19.05	.38	1142	43	22.89	.48	.19
12	36.65	.80	18.68	.37	1102	40	22.43	.46	.15
13	35.88	.77	18.32	.36	1065	37	21.99	.44	.10
14	35.15	.73	17.98	.34	1032	33	21.57	.42	.06
15	34.44	.71	17.65	.33	1002	30	21.16	.41	.01
16	33.75	.69	17.34	.31	975	27	20.77	.39	33.97
17	33.09	.66	17.03	.31	952	23	20.39	.38	.92
18	32.46	.63	16.74	.29	932	20	20.03	.36	.87
19	31.84	.62	16.46	.28	915	17	19.67	.36	.83
20	31.25	.59	16.19	.27	901	14	19.33	.34	.78
21	30.68	.57	15.92	.27			19.01	.32	.74
22	30.13	.55	15.67	.25			18.69	.32	.69
23	29.59	.54	15.42	.25			18.38	.31	.65
24	29.07	.52	15.18	.24			18.08	.30	.60
25	28.57	.50	14.95	.23			17.80	.28	.56
26	28.09	.48	14.73	.22			17.52	.28	.51
27	27.62	.47	14.51	.22			17.25	.27	.47
28	27.16	.46	14.30	.21			16.99	.26	.42
29	26.72	.44	14.10	.20			16.73	.26	.37
30	26.29	.43	13.90	.20			16.49	.24	.33
31	25.87	.42	13.70	.20			16.25	.24	.28
32	25.47	.40	13.52	.18			16.01	.24	.24
33	25.07	.40	13.34	.18			15.79	.22	.19
34	24.69	.38	13.16	.18			15.57	.22	.15
35	24.32	.37	12.99	.17			15.36	.21	.10
40	22.60		12.20				14.37		32.88
45	21.09		11.50				13.50		.65
50	19.75		10.88				12.73		.42

The coefficient of absorption of air given means the sum, $\lambda = 0.209 \beta_1 + 0.791 \beta_2$, and consequently refers (in theory) to ∞ volume of air used per 1 volume of water. From the above we see (as an example) that 1000 cc. of water of 15°C . when saturated at 15° and a barometric pressure of $B + \tau^*$ mm. with normal air, absorbs quantities of the three components which, when measured dry at B mm. pressure and 0°C ., amount :

I.	The oxygen to	$1000 \beta_1 \times .209 =$	7.196
II.	The nitrogen to	$1000 \beta_2 \times .791 =$	13.961
(I. + II.)	The two conjointly to	$1000 \lambda =$	21.16
III.	The carbonic acid to	$0.3 \beta_3 =$	0.30
Total,			21.46

Percentage of oxygen in I. + II. = $100 n_1^{**} = 34.01$.

ABSORPTION OF AIR BY SEA WATER. §

In the case *Sea Water*, the coefficients λ , α , β , and n_1 , assume different values λ' , α' , β' , n'_1 , governed by the equations :

$$\lambda' = \frac{.927 \ 31}{39 + t}$$

$$100 n'_1 = 34.40 - 0.0311 t.$$

$$\beta'_1 = \lambda' n'_1 \text{ for the Oxygen.}$$

$$\beta'_2 = \lambda' n'_2 \text{ for the Nitrogen.}$$

From these equations we have :

TABLE XVa. VALUES OF λ AND n_1 FOR SEA WATER.

t	1000 λ	100 n_1
-5°	27.27	34.56
0	23.78	.40
5	21.08	.24
10	18.92	.09
15	17.17	33.93
20	15.72	.78
25	14.49	.62
30	13.44	.47
35	12.53	.31

The corresponding Table in the Memoir (p. 175), goes from degree to degree, and includes the values $\lambda n_2 = \beta_2$ and the ratios $n_1 : n_2$

* τ stands for the tension of steam at 15° .

** The student will observe that from pp. 153 to 155 the symbols n_1 and n_2 have a different meaning from that assigned to n' and n'' on pp. 151 and 152.

§ Challenger Memoirs ; Physics and Chemistry, vol. I., pp. 173 and 174.

TABLE XVI. COEFFICIENTS OF ABSORPTION OF SOME GASES IN WATER OR ALCOHOL.

Taken from Bunsen, Gasometrische Methoden, Ed. II., pp. 384 to 387.

t	Nitrogen in Alcohol.	Hydrogen in Alcohol.	Carbonic Acid in Alcohol.	Carbonic Oxide in Water.	Nitric Oxide in Alcohol.	Nitrous Oxide in	
						Water.	Alcohol.
0°	.1263	.0693	4.330	.03287	.3161	1.305	4.178
1	.1259	.0691	4.237	.03207	.3126	1.261	4.109
2	.1255	.0690	4.147	.03131	.3093	1.217	4.041
3	.1251	.0688	4.059	.03057	.3060	1.175	3.974
4	.1248	.0687	3.974	.02987	.3029	1.135	3.909
5	.1244	.0685	3.891	.02920	.2999	1.095	3.844
6	.1241	.0684	3.811	.02857	.2969	1.058	3.781
7	.1237	.0683	3.733	.02796	.2941	1.021	3.719
8	.1234	.0681	3.657	.02739	.2913	0.986	3.659
9	.1231	.0680	3.584	.02686	.2887	0.952	3.599
10	.1228	.0679	3.514	.02635	.2861	0.920	3.541
11	.1225	.0677	3.446	.02588	.2836	0.889	3.484
12	.1222	.0676	3.381	.02544	.2813	0.859	3.428
13	.1219	.0675	3.318	.02504	.2790	0.830	3.373
14	.1217	.0674	3.257	.02466	.2769	0.803	3.320
15	.1214	.0673	3.199	.02432	.2748	0.778	3.268
16	.1212	.0671	3.144	.02402	.2728	0.754	3.217
17	.1210	.0670	3.091	.02374	.2709	0.731	3.167
18	.1208	.0669	3.040	.02350	.2692	0.709	3.119
19	.1206	.0668	2.992	.02329	.2675	0.689	3.071
20	.1204	.0667	2.947	.02312	.2659	0.670	3.025
21	.1202	.0666	2.903		.2644	0.653	2.981
22	.1201	.0665	2.863		.2631	0.636	2.937
23	.1199	.0664	2.825		.2618	0.626	2.894
24	.1198	.0663	2.789		.2606	0.608	2.853

TABLE XVI.—*Continued.*

t	Marsh Gas in		Ethylene in		Sulphuretted Hydrogen in	
	Water.	Alcohol.	Water.	Alcohol.	Water.	Alcohol.
0°	.0545	0.523	0.2563	3.595	4.371	17.89
1	.0533	0.520	0.2473	3.538	4.287	17.24
2	.0522	0.517	0.2388	3.482	4.205	16.61
3	.0510	0.514	0.2306	3.428	4.124	15.98
4	.0499	0.511	0.2227	3.375	4.044	15.37
5	.0489	0.509	0.2153	3.323	3.965	14.78
6	.0478	0.506	0.2082	3.273	3.887	14.19
7	.0467	0.503	0.2018	3.224	3.810	13.62
8	.0457	0.501	0.1952	3.177	3.735	13.07
9	.0447	0.498	0.1893	3.131	3.660	12.52
10	.0437	0.495	0.1837	3.086	3.586	11.99
11	.0428	0.493	0.1786	3.043	3.513	11.48
12	.0418	0.490	0.1737	3.001	3.442	10.97
13	.0409	0.488	0.1693	2.960	3.371	10.48
14	.0400	0.485	0.1652	2.921	3.301	10.00
15	.0391	0.483	0.1615	2.883	3.233	9.54
16	.0382	0.480	0.1583	2.846	3.165	9.09
17	.0374	0.478	0.1553	2.811	3.099	8.65
18	.0366	0.476	0.1528	2.777	3.033	8.23
19	.0358	0.473	0.1506	2.744	2.969	7.81
20	.0350	0.471	0.1488	2.713	2.905	7.42
21				2.683	2.843	7.03
22				2.655	2.782	6.66
23				2.628	2.722	6.30
24				2.602	2.662	5.96

In the following cases the coefficient of absorption is constant from 0° to 20° or 24° :—

Name of Gas.	Solvent.	Constant Value of β .	Interval of Temp.
Hydrogen,	Water	0.01930	0° to 24°
Oxygen,	Alcohol	0.2840	0° to 20°
Carbonic Oxide,	Alcohol	0.2044	0° to 20°

The following three Tables give the solubilities in water of some gases which do *not* obey Bunsen's law :—

TABLE XVII. AMMONIA AND SULPHUROUS ACID ACCORDING TO TH. H. SIMS.*

"G" signifies the number of grammes of gas absorbed by 1 gramme of water if the "dry" pressure of the gas = 760 mm. "Vol." stands for the volume of G grammes of (dry) gas at 0° and 760 mm.

AMMONIA.				SULPHUROUS ACID.		
t	G	t	G	t	G	Vol.
0°	.899	52°	.274	8°	.168	58.7
4	.809	56	.256	12	.142	49.6
8	.724	60	.238	16	.121	42.2
12	.646	64	.220	20	.104	36.4
16	.578	68	.202	24	.092	32.3
20	.518	72	.186	28	.083	28.9
24	.467	76	.170	32	.073	25.7
28	.426	80	.154	36	.065	22.8
32	.393	84	.138	40	.058	20.4
36	.363	88	.122	44	.053	18.4
40	.338	92	.106	48	.047	16.4
44	.315	96	.090	50	.045	15.6
48	.294	100	.074			

* Jahresb. for 1861, pp. 54 and 56.

TABLE XVIII. HYDROCHLORIC ACID ACCORDING TO H. E. ROSCOE AND W. DITTMAR.*

In this Table, G stands for the number of grammes of gas absorbed at t° by one gramme of water, if the *barometer* stands at 760 mm.

t	G	t	G	t	G	t	G
0°	.825	16°	.742	32°	.665	48°	.603
4	.804	20	.721	36	.649	52	.589
8	.783	24	.700	40	.633	56	.575
12	.762	28	.682	44	.618	60	.561

Aqueous hydrochloric acid of *any* strength, if boiled down briskly in a flask or retort (*not* in an open basin) under a given pressure P, loses or gains strength until its percentage of HCl has reached a certain value p which depends on P, or rather more directly on the boiling point determined by P. The residual acid then, on continued boiling, remains constant in composition. For $P = 0.760$ metres, involving the boiling point 110° , p is very exactly equal to 20.24. The following values p for other pressures are not so exactly determined. For

P = 0.1	0.5	1.0	1.5	2.0	2.5 metres.
p = 22.9	21.1	19.7	19.0	18.5	18.0

* Liebig's Ann. vol. 112, p. 327; Jahresb. for 1859, pp. 103 and 104.

THERMOMETRY.

TABLE I. CORRECTION TO BE "added" TO THE READING OF A MERCURY (CENTIGRADE) THERMOMETER TO OBTAIN THE CORRESPONDING INDICATIONS OF THE AIR THERMOMETER.*

Mercury Thermometer.	Correction.			
	I.	A.	B.	C.
0°	0.00			
5	-.01			
10	-.03			
15	-.07			
20	-.11			
25	-.12			
30	-.12			
35	-.10			
40	-.08			
54	-.04			
73	-.06			
82	+.04			
100	0.00			
110°		-.05	+.02	+.02
120		-.12	+.04	+.05
130		-.20	+.09	+.09
140		-.29	+.16	+.15
150		-.40	+.25	+.20
160		-.52	+.33	+.26
170		-.65	+.35	+.32
180		-.80	+.34	+.37
190		-1.01	+.32	+.35
200		-1.25	+.27	+.30
210		-1.53	+.18	+.25
220		-1.82	+.08	+.20
230		-2.16	-.02	+.15
240		-2.55	-.14	+.10
250		-3.00	-.26	-.05
260		-3.44	-.39	-.20
270		-3.90	-.50	-.38
280		-4.48	-.63	-.52
290		-5.10	-.88	-.80
300		-5.72	-1.21	-1.08

* Taken from Landolt and Börnstein's Tables.

I. Refers to thermometers made of modern Thüringian glass. Authority: Grunmach, *Metronomische Beiträge*, herausgegeben von der kaiserlichen Normal-Aichungs-Commission; No. 3, page 54; year 1881.

A. Results obtained by Regnault, with a thermometer, made of crystal from Choisy-le-Roy, containing about 34 per cent. of oxide of lead.

B. Regnault's result with a thermometer made of "*verre ordinaire*."

C. Mean of observations made by Crafts, with 15 thermometers, namely 7 each from Baudin and Alvergnyat with about 18 per cent. of oxide of lead, and one from Müller (Geisler's successor) in Bonn, made of German soda-glass.

After Crafts; *Comptes Rendus*, vol. 95, page 836; year 1882.

TABLE II. CORRECTION TO BE APPLIED TO THERMOMETER-READINGS IN BOILING-POINT DETERMINATIONS AND SIMILAR WORK, FOR THE LOWER TEMPERATURE OF THE THREAD OF MERCURY OUTSIDE THE MEDIUM.

T = Corrected temperature.

t = Temperature as read.

t' = Mean temperature of the outside-part of the thread of mercury.

n = Length of the outside-thread in degrees.

The correction, according to T. E. Thorpe, (Journ. Chem. Soc. vol. 37, p. 160; year 1880) is

$$\Delta = 0.000\ 143\ (t - t')\ n.$$

Log. 000 143 = .155 - 4. Three-place logarithms suffice.

Multiples of 0.000 143.

1 .000 143	2 .000 286	3 .000 429	4 .000 572	5 .000 715
6 .000 858	7 .001 001	8 .001 144	9 .001 287	

From a recent experimental investigation of E. Rimbach's (Ber. d. deutschen chem. Gesellschaft, year 1889, p. 3072), it appears that the *true* relations between Δ on the one hand and t, t' and n on the other demand, for their formulation, a more complex equation than the above, and in it the constants depend on the construction of the thermometer. They have for instance certain values for rod-thermometers (consisting of a plain thick capillary tube with the graduation on this tube) and others for "*enclosed*" thermometers, in which the stem of the thermometer is a thin capillary thread enclosed in a wide thin-walled envelope.

Rimbach gives very complete correction-tables for these two kinds of thermometers, based upon extensive experiments with instruments made of "*Jenaer Glas*."

According to Rimbach, Thorpe's formula, if applied to relatively long threads of mercury, gives corrections, *less* than the truth.

THERMOMETER SCALES.

The construction of the three scales known as the Centigrade or Celsius scale, the Fahrenheit scale, and the Réaumur scale respectively is seen from the following statement :—

	Centigrade.	Réaumur.	Fahrenheit.
Melting point of Ice,	0°	0°	32°
Boiling point of Water under a pressure of 760 mm. of Mer- cury of 0°C,*	100°	80°	212°
Number of degrees between the two fixed points,	100	80	180

In reducing a temperature from one of the three scales to another, imagine a thermometer carrying the respective two scales beside each other, note that, for *differences* of temperature,

$$5^{\circ}\text{C} = 4^{\circ}\text{R} = 9^{\circ}\text{F}.$$

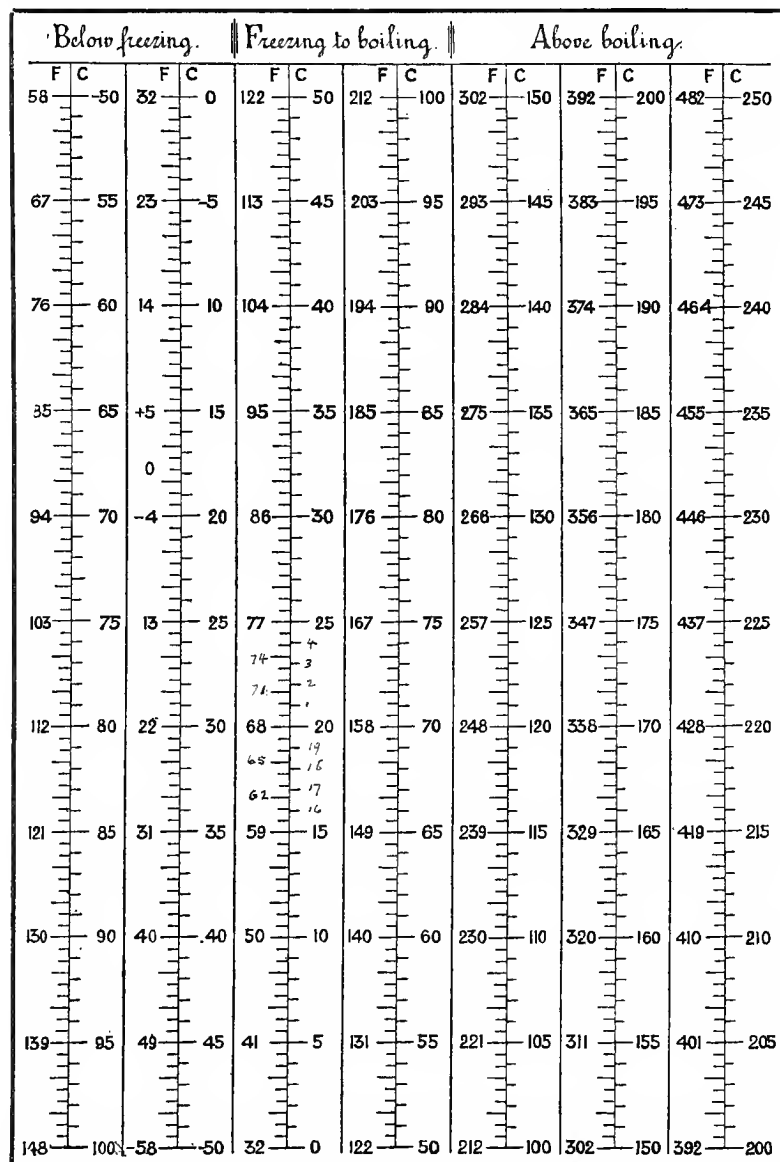
and the rest follows by itself.

* Reduced to lat. 45° and Sea-Level, strictly speaking. In Great Britain however the general practice is to define “212°F” as the temperature at which water boils, when the pressure of the atmosphere is equal to that of 30 inches (= 762 mm.) of mercury of 32°F., at the place of observation.

TABLE III. EQUIVALENT READINGS.

F.	C.	R.	F.	C.	R.	Equal Differences of Temperature.		
0	-17 $\frac{7}{9}$	-14 $\frac{2}{9}$	284	140	112	ΔF	ΔC	ΔR
-4	-20	-16	293	145	116			
+5	-15	-12	302	150	120			
+14	-10	-8	311	155	124	1	.56	.44
+23	-5	-4	320	160	128			
+32	0	0	329	165	132			
41	+5	+4	338	170	136	2	1.11	.89
50	10	8	347	175	140	3	1.67	1.33
59	15	12	356	180	144	4	2.22	1.78
68	20	16	365	185	148	5	2.78	2.22
77	25	20	374	190	152	6	3.33	2.67
86	30	24	383	195	156	7	3.89	3.11
95	35	28	392	200	160	8	4.44	3.56
104	40	32	401	205	164	9	5.00	4.00
113	45	36	410	210	168	ΔC	ΔF	ΔR
122	50	40	419	215	172			
131	55	44	428	220	176			
140	60	48	437	225	180	1	1.8	.8
149	65	52	446	230	184			
158	70	56	455	235	188			
167	75	60	464	240	192	2	3.6	1.6
176	80	64	473	245	196	3	5.4	2.4
185	85	68	482	250	200	4	7.2	3.2
194	90	72	491	255	204	5	9.0	4.0
203	95	76	500	260	208	ΔR	ΔC	ΔF
212	100	80	509	265	212			
221	105	84	518	270	216			
230	110	88	527	275	220	1	1.25	2.25
239	115	92	536	280	224	2	2.50	4.50
248	120	96	545	285	228	3	3.75	6.75
257	125	100	554	290	232	4	5.00	9.00
266	130	104	563	295	236			
275	135	108	572	300	240			

The following Diagram representing a Double Thermometer Scale, Fahrenheit degrees on the left side and Centigrade on the right, competes with Table III. as far as these two scales are concerned ; but, to those who have learned to guess tenths of degrees, offers the advantage of giving the desired result without interpolation—apart from the reading error, which however needs not exceed 0.1°C or 0.2°Fahr.



CORRECTIONS AND ADDITIONS.

ATOMIC WEIGHTS, page 1.

LITHIUM.—Since the publication of my memoir I have caused Mr. J. F. Ness to try and check my result by analyses of the chloride. His results so far agree better with $\text{Li} = 7.00$ than with my number, 6.89. We are engaged in fresh analyses of the carbonate by means of an improved method in order to explain the discrepancy. Meanwhile analyses had better be calculated after the value 7.00.

HYDROGEN.—Since the completion of the MS. for this volume I undertook, conjointly with Mr. J. B. Henderson, a critical experimental enquiry on Dumas' and on Erdmann and Marchand's determinations of the constant $\text{H}_2 : \text{O}$, which ultimately developed into a re-determination of its value. Our principal results are as follows:—(1) Dumas' results, if taken conjointly with Erdmann and Marchand's and properly interpreted, assign to the ratio $\text{O} : \text{H}$ the value 15.996, or practically, 16.00. But (2) Dumas, it seems, forgot to reduce his water-weights to the vacuum, while Erdmann and Marchand lost some of their water by employing only chloride of calcium as a dehydrator for the gas which left their water-receptacle. Allowing for the respective errors we arrived at values lying at about 1.004 and 1.006 respectively. (3) Our own (final) series of (ten) syntheses gave values lying between 1.0073 and 1.0114; the mean of the ten values was 1.009118; probable error of a single observation = ± 0.00087 ; of the mean = $\pm .00027$. But, although all the ten experiments were successful, we had reason to attach greater weight to certain seven of the ten results than to the other three. These seven results varied from $\text{H} = 1.00888$ to 1.00949; mean of the seven numbers = 1.008808. But all our numbers as given so far are liable to a small correction for the unavoidable presence of occluded hydrogen in the reduced copper produced. The correction which we applied was based upon experimental determinations of the occluded hydrogen in a previous series. Referring to our memoir for details, we confine ourselves here to giving the number which we adopted finally as being, in our opinion, the most probable. It is $\text{H} = 1.008473$.

But ours is only *one* of quite a series of recent attempts to fix the value of this constant; the following table gives the names and results of our principal predecessors.

TAKING $O = 16$.

I. Lord Rayleigh found	$H = 1.006\ 92$
II. Cooke and Richards	$1.008\ 25$
III. W. A. Noyes	$1.007\ 17$
IV. Dittmar and Henderson ; their adopted number	$1.008\ 47$
(IVa. D. & H.; the mean of the ten experi- ments corrected -	$1.008\ 78)$
V. E. H. Keiser	$1.003\ 18$

The first four results, as we see, agree very fairly with one another ; their mean, 1.00702 , indeed, does not differ from any of the numbers I. to IV. by more than its possible error ; but Keiser's value is so far below any of the rest that either it or they must be infected with an unobserved error. In these circumstances one feels inclined to adopt that mean value 1.00702 as being, at present, probably the closest obtainable approximation to the truth. Considering, however, that of the four methods used, Dittmar and Henderson's is by far the easiest of execution, I prefer to adopt their result, which practically may be put down as

$$H = 1.0085$$

as the most probable value. But, as I do so, I ought, in strictness, to correct all those of my formula-values, analytical factors, and gasometric constants, which depend on the atomic weight of hydrogen. In practice, however, it suffices to apply the correction to those numbers whose inherent uncertainty is not greater than the correction itself, and these form only a small minority. All the *necessary* corrections are found below under their respective headings.

VALUES OF THE "PROBABILITY-INTEGRAL," &c., page 51.

As a supplement thereto, we will give the formulae for the calculation of the probable error " r " of a single determination in, and of the probable error r_0 of the mean of, a series of direct determinations of an unknown quantity.

If the differences between the mean x_0 and the individual results x' , x'' , x''' . . . are $x' - x_0 = v'$; $x'' - x_0 = v''$; $x''' - x_0 = v'''$, etc., and the number of the determinations is n , we have

$$r = \pm .6745 \sqrt{\frac{\sum v^2}{n-1}} \quad \text{Log. Const.} = .8290 - 1$$

$$\text{and } r_0 = \frac{r}{\sqrt{n}} = \pm .6745 \sqrt{\frac{\sum v^2}{n(n-1)}}$$

or approximately,

$$r = \pm \frac{.8453 \sum v}{\sqrt{n(n-1)}}$$

$$\text{Log. Const.} = .9270 - 1$$

and $r = \frac{r}{\sqrt{n}}$ as before.

In the approximate formula, $\sum v$ stands for the *arithmetical* sum of the numbers v . Thus, for instance, if $v' = -1$, $v'' = -2$, $v''' = +5$, then $\sum v = 8$, and *not* $+2$.

Since the values r and r_0 , whether calculated by one of the formulæ or the other, cannot be more than guesses at naturally indeterminate quantities, it is permissible to substitute $n - 0.5$ for $\sqrt{n(n-1)}$.

Examples. $\sqrt{3 \times 2} = 2.449$; $\sqrt{4 \times 3} = 3.464$; $\sqrt{5 \times 4} = 4.472$;
 $\sqrt{6 \times 5} = 5.477$.

The approximation, of course, becomes the better the greater n . Thus,
 $\sqrt{13 \times 12} = 12.490$; $\sqrt{20 \times 19} = 19.494$.

TABLE OF SQUARE-ROOTS.

n	$\sqrt{n}$	n	$\sqrt{n}$	n	$\sqrt{n}$	n	$\sqrt{n}$
1	1.0000	26	5.0990	51	7.1414	76	8.7178
2	1.4142	27	5.1962	52	7.2111	77	8.7750
3	1.7321	28	5.2915	53	7.2801	78	8.8318
4	2.0000	29	5.3852	54	7.3485	79	8.8882
5	2.2361	30	5.4772	55	7.4162	80	8.9443
6	2.4495	31	5.5678	56	7.4833	81	9.0000
7	2.6458	32	5.6569	57	7.5498	82	9.0554
8	2.8284	33	5.7446	58	7.6158	83	9.1104
9	3.0000	34	5.8310	59	7.6811	84	9.1652
10	3.1623	35	5.9161	60	7.7460	85	9.2195
11	3.3166	36	6.0000	61	7.8102	86	9.2736
12	3.4641	37	6.0828	62	7.8740	87	9.3274
13	3.6056	38	6.1644	63	7.9373	88	9.3808
14	3.7417	39	6.2450	64	8.0000	89	9.4340
15	3.8730	40	6.3246	65	8.0623	90	9.4868
16	4.0000	41	6.4031	66	8.1240	91	9.5394
17	4.1231	42	6.4807	67	8.1854	92	9.5917
18	4.2426	43	6.5574	68	8.2462	93	9.6437
19	4.3589	44	6.6332	69	8.3066	94	9.6954
20	4.4721	45	6.7082	70	8.3666	95	9.7468
21	4.5826	46	6.7823	71	8.4261	96	9.7980
22	4.6904	47	6.8557	72	8.4853	97	9.8489
23	4.7958	48	6.9282	73	8.5440	98	9.8995
24	4.8990	49	7.0000	74	8.6023	99	9.9499
25	5.0000	50	7.0711	75	8.6603	100	10.0000

FORMULA VALUES.

HYDROGEN, page 55.

The corrected values are :—

	F (O=16.)	Log. F.
Hydrogen,	1.0085	.00368

Multiples.

1.0085; 2.0170; 3.0255; 4.0340; 5.0425; 6.0510; 7.0595;
8.0680; 9.0765; 10.0850.

Water, H ₂ O	18.017	.25568
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VALUES OF $n \times 18.017$.

n	Multiple.	Log. Multiple.*	n	Multiple.	Log. Multiple.*
1	18.02	.25575	16	288.27	.45980
2	36.03	.55666	17	306.29	.48613
3	54.05	.73280	18	324.31	.51096
4	72.07	.85775	19	342.32	.53443
5	90.09	.95468	20	360.34	.55671
6	108.10	.03383	21	378.36	.57790
7	126.12	.10078	22	396.37	.59810
8	144.14	.15878	23	414.39	.61741
9	162.15	.20992	24	432.41	.63590
10	180.17	.25568	25	450.43	.65363
11	198.19	.29708	26	468.44	.67065
12	216.20	.33486	27	486.46	.68705
13	234.22	.36962	28	504.48	.70284
14	252.24	.40181	29	522.49	.71808
15	270.26	.43178	30	540.51	.73280

* The "Logg." are the logarithms of the multiples as they stand in the table, not the logarithms of the exact multiples of 18.017.

LITHIUM, page 56.

Adopt the values given under Li = 7.00.

NITROGEN, page 57.

The corrections embodied in the following numbers are very slight; yet we thought we had better effect them. For H = 1.0085, we have:—

	F.	Log. F.
NH ₃	17.071	.23226
NH ₄	18.080	.25720
(NH ₄) ₂ O	52.160	.71734
NH ₄ Cl	53.534	.72863

ANALYTICAL FACTORS.

LITHIUM, page 63.

Adopt the numbers given under Li = 7.00.

NITROGEN, page 64.

For H = 1.0085, we have :—

a	b	b : a = f	Log. f
N	NH ₃	1.21536	.08471
NH ₃	N	0.82280	.91529
2NH ₃	(NH ₄) ₂ O	1.52774	.18405
(HN ₄) ₂ O	2NH ₃	0.65456	.81595
NH ₄ Cl	NH ₃	0.31888	.50363
2NH ₄ Cl	(NH ₄) ₂ O	0.48717	.68768
PtCl ₆ (NH ₄) ₂	2NH ₃	0.07683	.88553
PtCl ₆ (NH ₄) ₂	(NH ₄) ₂ O	0.11738	.06958
Pt	2NH ₃	0.17464	.24214
Pt	(NH ₄) ₂ O	0.26680	.42619

EMPIRICAL FACTORS, page 67.

*(c) Determination of Ammonia given as Sal-Ammoniac.**For H = 1.0085, we have :—*

Given.	Wanted.	f.	Log. f.
PtCl ₆ (NH ₄) ₂	2NH ₄ Cl	0.2389*	.37822
PtCl ₆ (NH ₄) ₂	2NH ₃	0.07618	.88185
PtCl ₆ (NH ₄) ₂	2N	0.06268	.79714
Pt	2NH ₄ Cl	0.5459*	.73711
Pt	2NH ₃	0.17408	.24074
Pt	2N	0.14323	.15603

* Independent of the value assumed for H, because directly determined.

THE METRIC AND BRITISH SYSTEM OF UNITS.

OTHER RELATIONS, page 73.

Additional entry :—

1 grain = 0.064 799 grammes. Log. = .811 568 - 2.

CORRECTIONS OF WEIGHINGS.

TABLE I., page 103.

This table is inconvenient, the intervals being too large to admit of mental interpolation; we therefore here substitute another edition, which virtually gives the values of D for every tenth of a degree.

WEIGHT, IN GRAMMES, OF 1 LITRE OF (DRY) AIR CONTAINING 0.04 % BY VOLUME OF CARBONIC ACID, supposing the air to be measured at t° C and 760 mm. of mercury of 0° C; page 103.

t	Weight of 1 Litre D.	Diff. (Neg.)	P. P.		t	Weight of 1 Litre D.	Diff. (Neg.)	P. P.	
			Δt	ΔD				Δt	ΔD
0°	1.2943				15°	1.2267			
1	1.2895	48	0.1°	5	16	1.2225	42	0.1°	4
2	1.2848	47	0.2	9	17	1.2182	43	0.2	8
3	1.2802	46	0.3	14	18	1.2140	42	0.3	13
4	1.2755	47	0.4	19	19	1.2099	41	0.4	17
			0.5	23				0.5	21
			0.6	28				0.6	25
			0.7	33				0.7	29
5	1.2709	46	0.8	37	20	1.2057	42	0.8	33
6	1.2664	45	0.9	42	21	1.2016	41	0.9	38
7	1.2618	46	1.0	46.4	22	1.1975	41	1.0	41.7
8	1.2573	45			23	1.1935	40		
9	1.2529	44	Δt	ΔD	24	1.1895	40	Δt	ΔD
			0.1°	4				0.1°	4
10	1.2484	45	0.2	9	25	1.1855	40	0.2	8
11	1.2440	44	0.3	13	26	1.1815	40	0.3	12
12	1.2397	43	0.4	17	27	1.1776	39	0.4	16
13	1.2353	44	0.5	22	28	1.1736	40	0.5	20
14	1.2310	43	0.6	26	29	1.1697	39	0.6	24
			0.7	31				0.7	28
			0.8	35				0.8	32
			0.9	39				0.9	35
15	1.2267	43	1.0	43.7	30	1.1659	38	1.0	39.4

$$D \text{ at } 760 \pm p = \left\{ D \text{ at } 760 \right\} \left\{ 1 \pm \frac{p}{760} \right\} = \left\{ D \text{ at } 760 \right\} \left\{ 1 \pm .0013158 p \right\}$$

Multiples of 0.001 315 8.

1 .001 316	2 .002 632	3 .003 947	4 .005 263	5 .006 579
6 .007 895	7 .009 211	8 .010 526	9 .011 842	10 .013 158

LOGARITHMIC TABLE.

t	Log. D.	Diff. (Neg.)	P. P.		t	Log. D.	Diff. (Neg.)	P. P.	
			Δt	$\Delta \log.$				Δt	$\Delta \log.$
0°	.11202		0.1°	16	15°	.08875		0.1°	15
1	.11043	159	0.2	31	16	.08724	151	0.2	30
2	.10885	158	0.3	47	17	.08573	151	0.3	45
3	.10727	158	0.4	63	18	.08424	149	0.4	60
4	.10569	158	0.5	79	19	.08274	150	0.5	75
			0.6	94				0.6	90
			0.7	110				0.7	105
5	.10413	156	0.8	126	20	.08126	148	0.8	120
6	.10256	157	0.9	142	21	.07977	149	0.9	134
7	.10101	155	1.0	157.3	22	.07829	148	1.0	149.4
8	.09945	156			23	.07682	147		
9	.09791	154	Δt	$\Delta \log.$	24	.07535	147	Δt	$\Delta \log.$
			0.1°	15				0.1°	15
10	.09637	154	0.2	31	25	.07389	146	0.2	29
11	.09483	154	0.3	46	26	.07243	146	0.3	44
12	.09330	153	0.4	61	27	.07098	145	0.4	58
13	.09178	152	0.5	76	28	.06953	145	0.5	73
14	.09026	152	0.6	92	29	.06809	144	0.6	87
			0.7	107				0.7	102
			0.8	122				0.8	116
			0.9	138				0.9	131
15	.08875	151	1.0	152.9	30	.06665	144	1.0	145.3

For the value D' corresponding to the pressure P' mm. we have :—

$$\text{Log. } D' = \text{Log. } D + \text{Log. } P' + .119 \ 19 - 3.$$

GASOMETRY.

Correction for the Gravity at the place of Observation, page 130.

For the calculation of the relative gravity at a place from its latitude ϕ and elevation H above the level of the sea, Lasch, in his memoir, gives a formula, consisting of two factors of which the second is based on the assumption that G is inversely proportional to the square of $R + H$, where R stands for the earth's radius. By substituting for R its value in metres, and effecting an easy transformation, I deduced from it the formula

$$G = (1 - .002 \ 5935 \cos. 2 \phi) (1 - .000 \ 000 \ 3142 \ H)$$

which stands in the context, and I used it for calculating the values of G given in Table IV., page 131, although I was aware that it does not agree with the formula

$$G = (1 - .002 \ 59 \cos. 2 \phi) (1 - .000 \ 000 \ 196 \ H)$$

which Landolt and Börnstein give (*autoritate* Broch) in their "*Tabellen.*" My reason was, that while I understood the meaning of Lasch's factor

for the influence of H, I did not know the basis of Broch's. Through the courtesy of Professor Börnstein, however, I am now enabled to explain the difference. Our (Lasch's) formula, obviously, is perfectly correct only, if the place of observation lies in the *air*, H metres above the plane surface of the earth which is supposed to coincide with the surface of the sea; Broch's on the other hand allows for the attraction of the strata of earth between the sea-level and the place. I am ignorant of Broch's mode of calculation, but, having no doubt about its soundness, I give in the following Table the values of G for some of the places enumerated in my Table IV., page 131, calculated by Broch's formula:—

RELATIVE VALUE OF GRAVITY AT SOME STATIONS.

STATION.	VALUE OF GRAVITY, G.	
	G at 45°, and H=0, taken as = 1.	G in Paris = 1.
	$1 + \gamma$	$1 + \delta$
Lat. 45° and Sea-Level	1.000 000	.999 666
Rome,	.999 714	.999 380
Milan,	1.000 016	.999 682
Paris,	1.000 334	1.000 000
London, -	1.000 575	1.000 241
Berlin,	1.000 664	1.000 330
Edinburgh-Glasgow,	1.000 952	1.000 618
Aberdeen,	1.001 064	1.000 730

The values $1 + \delta$, as we see, agree almost absolutely with those given in the table in the context.

PHYSICAL CONSTANTS OF A NUMBER OF GASES, page 137.

As shown by Lord Rayleigh, Regnault's gas-densities are infected with an error owing to his having failed to allow for the expansion which his globes suffered when, after having been tared in an evacuated condition, they were filled with the respective gases at the prevailing pressure of the atmosphere. In the case of nitrogen and of gases denser than nitrogen the error does not come to much, but it assumes an appreciable value in the case of hydrogen.

According to Rayleigh's* own determinations (in which the expansion of the globes was determined by experiments with water and allowed for) the specific gravity of oxygen on the hydrogen-scale is = 15.884 under ordinary atmospheric conditions. From this number and Regnault's value for oxygen and air I have calculated the entries marked "Rayleigh" in the supplement to Table X., page 137, given on page 174.

WEIGHT IN GRAMMES OF 1 LITRE OF GAS IF MEASURED AT t° AND A PRESSURE OF P MM. OF MERCURY OF 0° ; page 138.

Name of Gas.	Regnault's Laboratory.	Edinburgh-Glasgow.
Hydrogen, by Rayleigh's D_0 and Regnault's α	$.032\ 350 \frac{P}{273.13 + t}$ Log. = .509 87 - 2	$.032\ 370 \frac{P}{273.13 + t}$ Log. = .510 14 - 2

* Proceedings of the Roy. Soc., vol. 43 (year 1888), p. 362.

PHYSICAL CONSTANTS OF A NUMBER OF GASES.

NAME AND FORMULA.	Specific Gravity under Atmospheric Conditions.			Weight of 1 Litre in Grammes, if t = 0° and P = 760 mm. of Mercury of 0°.				
	Air = 1	Oxygen = 16	Hydrogen = 1	Lat. 45° and Sea-Level.	Regnault's Laboratory.	London.	Berlin.	Edinburgh-Glasgow $\delta = .000618$
HYDROGEN.								
Rayleigh, - - -	.069 606	1.007 30*	1	.089 985	.090 015	.090 037	.090 045	.090 071
By Dittmar and Henderson's determination of H ₂ : O § - -	.069 687	1.008 47	1	.090 090	.090 120	.090 141	.090 149	.090 175
Regnault, for comparison, - -	.069 268	1.002 4	1	...	.089 578	...	...	...
OXYGEN.								
Rayleigh, - - -	...	...	15.884*	...	...	...	...	...
Dittmar and Henderson, - -	...	...	15.866	...	...	...	...	...
Regnault, - - -	...	...	15.962	1.429 33	1.429 803	1.430 15	1.430 28	1.430 69
AIR.								
By Regnault's determinations, -	1.000 000	14.471 4	14.366	1.292 78	1.293 204	1.293 52	1.293 63	1.294 00

* By direct determination. § Assuming that water contains 2 vol. of hydrogen for 1 of oxygen; but according to Scott (see Rayleigh's Memoir) 1 vol. of oxygen combines with 1.9965 vol of hydrogen. || Regnault-Rayleigh.

TABLE FOR THE REDUCTION OF GASOMETRIC CONSTANTS TO ONE ANOTHER.

For $O_2 : H_2 = 16 : 1.0085$, we have :—

A.—*Given: The Molecular Weight, O = 16; page 142.*

Wanted.	Formula.
(2) Specific Gravity, S_H ; Hydrogen = 1	$S_H = 0.49579 M.$ $\text{Log.} = .69529 - 1$

B.—*Wanted: The Molecular Weight, O = 16; page 143.*

Given.	Formula.
(2) Specific Gravity, S_H ; Hydrogen = 1	$M = 2.0170 S_H;$ $\text{Log.} = .30471$

D.—*Change of Standard for Specific Gravity S; page 144.*

Given S in reference to	Wanted S in reference to	Multiply by f	
		f	Log. f
Oxygen = 16	Hydrogen = 1	0.99157	.99632 - 1
Air = 1	Hydrogen = 1	14.349	1.15684
Hydrogen = 1	Air = 1	0.069689	.84317 - 2
Hydrogen = 1	Oxygen = 16	1.0085	.00368

OTHER WORKS BY THE SAME AUTHOR.

I.—Report on the Composition of Ocean Water:

Part of "The Physics and Chemistry of the Voyage of H.M.S. *Challenger*." Part I. Printed for Her Majesty's Stationery Office. Sold by Adam and Charles Black, and Douglas & Foulis, Edinburgh; and others. 1884.

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Douglas & Foulis, Edinburgh, 1876. TABLES belonging thereto, same publishers, 1877.

III.—Analytical Chemistry:

A Series of Laboratory Exercises, constituting an Elementary Course of Qualitative Analysis. W. & R. Chambers, Edinburgh; 2nd edition; 1886.

IV.—Exercises in Quantitative Chemical Analysis:

With a Short Treatise on Gas Analysis. William Hodge & Co., Glasgow; Williams & Norgate, London and Edinburgh. 1887.

EXERCISES IN THE PREPARATION OF ORGANIC COMPOUNDS.

By Prof. Emil Fischer, Würzburg. Translated, with permission of the Author, from the second German edition, by Archibald Kling, F.I.C., with a Preface by W. Dittmar, LL.D., F.R.S.S. L. and E. William Hodge & Co., Glasgow; Williams & Norgate, London and Edinburgh. 1889.

